# A Structural Model of Productivity, Uncertain Demand, and Export Dynamics*

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**Abstract**

This paper develops and estimates a dynamic model of exporting to quantify how productivity and uncertain foreign demand separately influence firms' export participation. In this model, firms face uncertainty about their own foreign demand, and they update their beliefs based on individual export transactions according to Bayes' rule. I estimate the model using data on firm-level production and transaction-level exports to Germany in the Chinese ceramics and glass industry. The empirical results show substantial heterogeneity in productivity and demand belief. For experienced firms, productivity is the major driving force of export participation. In contrast, for potential entrants, demand learning plays a more important role. A counterfactual exercise suggests that trade cost reduction has a significant impact on stimulating the export participation of potential entrants. Importantly, more than half of the participation increase is attributed to firms' endogenous reaction to demand learning in the foreign market.

**Keywords:** *demand uncertainty, Bayesian learning, productivity, export participation*

**JEL Classification:** F14, D83

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# 1 Introduction

The international trade literature has documented productivity and learning about uncertain foreign demand as driving forces of firms' export decisions. Many studies (e.g., Melitz, 2003; Bernard et al., 2003; Aw et al., 2011; Eaton et al., 2011) show that firms with higher productivity self-select to export. More recently, a growing strand of trade models emphasizes the role of demand uncertainty and learning in export markets (e.g., Rauch and Watson, 2003; Freund and Pierola, 2010; Albornoz et al., 2012; Nguyen, 2012; Eaton et al., 2013; Timoshenko, 2015; Berman et al., forthcoming). However, without incorporating the demand learning process and productivity evolution in the same framework, these papers are not able to investigate empirically their individual roles in export decisions.

In this paper, I develop and estimate a dynamic structural model of exporting to disentangle how firms' export decisions are separately driven by the two forces. This is a single-agent model with an infinite horizon. The firm faces uncertainty about its own foreign demand and gradually learns about it from exporting according to Bayes' rule (in the spirit of Jovanovic, 1982). Specifically, the size of each individual export transaction updates the firm's expectation about its own foreign demand. The demand uncertainty gradually resolves as transactions accumulate. At the same time, the firm's productivity evolves over time. Both demand belief and productivity influence the firm's export decision. In the estimation, a novelty is that I use data on firm-level production and transaction-level exports to identify the roles of the two driving forces.

The recent literature (e.g., Foster et al., 2008; Pozzi and Schivardi, 2012) documents demand heterogeneity as a determinant of firm behavior. In the export context, Roberts et al. (forthcoming) find substantial heterogeneity in the demand and cost dimensions, with demand being more dispersed. My paper builds on the insight that foreign demand is heterogeneous across firms, but it departs from the literature to model firms' uncertainty about their own foreign demand. In this model, it is the firm's demand belief (rather than the demand itself) that directly influences the firm's export decision. As a result, the export decision and the demand learning process are endogenously related. The firm makes the export decision based on its current demand belief, and, in turn, the export experience updates the demand belief, which affects future export decisions.

However, the assessment of the role of demand learning is potentially biased if the evolution of

productivity is ignored. Productivity has been long-recognized as a source of heterogeneity in the studies of firm performance (e.g., Hopenhayn, 1992; Baily et al., 1992; Baldwin and Gorecki, 1998; Bartelsman and Doms, 2000; Irarrazabal and Opromolla, 2006; Luttmer, 2007; Syverson, 2011) as well as export decisions (e.g., Helpman et al., 2004; Arkolakis, 2011). Eaton et al. (2011) demonstrate that over half of the variation across firms in market entry can be attributed to productivity heterogeneity. In the time dimension, Aw et al. (2011) find that research and development (R&D) and exporting have a positive effect on firms' future productivity, which in turn drives more firms to self-select into both activities. Based on the importance of productivity, this paper explicitly models its evolution to avoid attributing productivity changes to the role of demand learning.

It is an empirical challenge to separate the role of the learning process from the effect of productivity evolution, because neither is observable to researchers. Moreover, the widely-used productivity measure estimated from total revenue and inputs contains both physical production efficiency and demand (Foster et al., 2008). To address these issues, I model demand learning as a process of learning about the relative difference between the firm's domestic demand and foreign demand. Given that most firms have operated in the domestic market for years, it is plausible to assume that firms have no uncertainty about their domestic demand.[1] Thus, firms' learning about foreign demand is equivalent to learning about the difference between domestic and foreign demands. This enables me to directly use a measure of domestic revenue-based productivity, which can be estimated using domestic revenue and inputs. Such a productivity measure influences export participation by giving the firm a reference profitability in the foreign market based on its demand in the domestic market. The firm's belief about the difference between foreign and domestic demands (i.e., demand belief) adjusts the reference toward the expected profitability in the foreign market. Variation in demand belief comes from export experience. Conditional on exporting, the revenue from each individual export transaction relative to domestic sales reflects a noisy signal of the difference in demand between the two markets. The firm uses this signal to update its demand belief following Bayes' rule, which influences the firm's export participation in the next period.

As such, the identification and estimation strategy relies on two data sources. The first source is the Chinese Annual Survey of Industrial Firms. It provides firm-level production information

[1] The opportunity for exporting opened up for most firms after China's accession to the World Trade Organization (WTO) in 2001. Prior to that, most of them had little experience. In contrast, the median firm age was seven years in 2001 (as a measure of domestic experience) in the data used in this study.

and enables me to estimate a measure of domestic revenue-based productivity that varies across firms and over time. The second source is the Chinese Customs Transactions data. The individual transaction-level exports from this source contain information on how Bayesian learning occurs. This data allows me to construct demand signals and use them to update firms' demand beliefs in each year. The integration of the two data sources provides firm-time variation in productivity and demand belief separately. A model with only productivity heterogeneity predicts that more productive firms export. In contrast, in my model with two-dimensional heterogeneity, firms with higher demand expectation may also export, even if their productivity is not high. With productivity and demand beliefs controlled, differences in export participation patterns help to identify the role of each driving force. The estimation is a practical application of mathematical program with equilibrium constraints (MPEC) to the dynamic discrete choice model of export participation. MPEC was developed by Su and Judd (2012) and Dubé et al. (2012). It was further implemented in Barwick and Pathak (2015). It allows me to avoid estimation inconsistency and reduces the computational burden of the high-dimension state dynamic model.

I estimate the model using data on production and exports to Germany in the Chinese ceramics and glass industry during 2000 to 2006. Firms may have different export experience prior to the sample period. Thus, I divide them into two groups and allow them to hold different prior beliefs. Specifically, I distinguish potential entrants and experienced firms in the German market according to the firms' initial export status in the sample period.[2] Within each group, I find substantial heterogeneity in productivity and demand belief. Across groups, experienced firms have higher productivity and demand expectation, and they face less uncertainty, compared with potential entrants. The dominant difference lies in demand expectation and uncertainty. Because productivity is estimated from domestic sales, the results suggest that foreign demand and its uncertainty (rather than domestic performance) serve as the main predictors of participation in the export market. These results add to the recent literature that documents demand as a determinant of export decisions beyond productivity (e.g., Eckel et al., 2015; Hottman et al., 2016; Aw and Lee, 2017; Roberts et al., forthcoming).

I use the estimated model to conduct counterfactual exercises and quantify the individual roles

[2]For example, a firm that exported in 2000 is classified as an experienced firm. Such grouping is fixed over the sample period. This is essentially to use the export status in the initial year of the sample period to approximate the experience of a firm in the pre-sample period. I discuss the approximation and its robustness in Section 5.2.

of the two driving forces. I consider two scenarios, with the evolution of productivity or demand learning removed from the model. I compare the results simulated from the two scenarios against the results generated from the full model. For experienced firms, I find that productivity evolution is the major driving force of export participation. But for potential entrants, the learning process plays a more important role. Overall, the results suggest the importance of learning about unobserved demand at the time of the export decision. This complements Dickstein and Morales (2015), who measure the observable information set of firms when they make export decisions.

Moreover, I find that entrants who received high demand signals continued exporting in the foreign market. In an environment of demand uncertainty, this implies that a policy that reduces the cost of trade may have an indirect effect on boosting export participation. This is in addition to its direct impact on enabling less productive (or lower demand) firms to export. The indirect effect comes from firms' endogenous reaction to what they learn in the foreign market. This is similar to Arkolakis et al. (2017), who show that fixed cost subsidies to young firms in an uncertain environment can prevent early exit from the market. Complementing their work, I consider productivity and demand learning in the export context. I find that the indirect effect accounts for more than half of the full impact after the cost of trade is reduced by 20 percent. Nonetheless, the indirect effect slowly declines over time and is almost caught up by the direct effect in the long run.

This paper contributes to the vibrant literature on firm exporting and Bayesian learning of foreign demand.[3] In particular, Timoshenko (2015) explains export product switching in a model of learning about appeal indices. Berman et al. (forthcoming) document demand learning as an important driver of post-entry export decisions by using detailed French exporter-level data. I add to the literature by estimating a dynamic model and quantitatively evaluating the roles of demand learning and productivity evolution in determining export participation.

The remainder of the paper is organized as follows. Section 2 develops a structural model of export dynamics with productivity evolution and demand learning. Section 3 describes the identification and estimation strategy. Section 4 describes the data for the estimation. The results are discussed in Section 5. Section 6 conducts counterfactual exercises. I conclude in Section 7.

[3]This paper also fits into broader empirical literature on the study of uncertainty and learning in structural models. For example, see Ackerberg (2003) for a dynamic learning model to study the "informative" and "prestige" effects of advertising; Abbring and Campbell (2003) for a model of firm growth, learning, and survival for bars in Texas; and Crawford and Shum (2005) for a study of the effects of uncertainty and learning for pharmaceutical drugs.

# 2 Model

This section develops a dynamic model of exporting in which firms face uncertainty about their own demand in foreign markets. I first provide an overview of the model to explain the timing and key assumptions, and the details of the model follow.

## 2.1 Overview

Consider an industry with $I$ firms (indexed by $i$) and an infinite-period horizon. There are a total of $J$ foreign markets (indexed by $j$) in addition to the domestic market (denoted as $D$). Firms are heterogeneous in demand and productivity. For each firm, its demand is different across markets and its productivity evolves exogenously over time.[4]

In each period $t$, each firm $i$ makes decisions about production, pricing, and export participation in each foreign market, in addition to serving the domestic market.[5] Two types of information are crucial for the firm to make these decisions. The first one is its productivity, and the second is its demand in each market. At the beginning of each period $t$, the firm observes its domestic demand and current productivity. However, in contrast to the recent literature (e.g., Roberts et al., forthcoming), I assume the firm's demand in a foreign market is unknown to the firm and the firm holds a belief regarding it. Given productivity, it is the current belief regarding the demand in the foreign market (rather than the demand itself) that directly influences the firm's decision to export.

I assume that the firm's export decisions for different markets are independent. If the firm exports to market $j$ in period $t$, then it receives a number of orders from customers in that market after paying export costs. At the end of this period, the firm can learn about its demand in market $j$, by comparing the size of each order with its domestic sales. Specifically, the firm's demand belief for market $j$ is updated at the end of period $t$ according to Bayes' rule. However, if the firm decides not to export in market $j$, then the belief regarding the demand in that market will not be updated at the end of period $t$.

Several points need to be clarified. First, the firm's learning processes in different markets are

[4] If firms endogenously choose productivity, for example by investing in R&D, then more productive firms are more likely to conduct R&D (Doraszelski and Jaumandreu, 2013). The assumption of exogenous productivity evolution will attribute the impact from R&D to the high persistence of productivity. I abstract from endogenous productivity evolution in this paper because of the lack of R&D data.

[5] I assume that firms always serve the domestic market and focus on their decisions to enter and exit foreign markets.

assumed to be independent. More precisely, I assume the firm does not apply what it has observed in one market to another market. Some papers relax this assumption, but they either consider a finite-horizon model (e.g., Albornoz et al., 2012) or one-shot learning (e.g., Nguyen, 2012).[6] Further, they do not estimate a dynamic model or separate the roles of productivity evolution and demand learning. Second, it is feasible to consider that productivity evolution is affected by export participation (i.e., learning by exporting). Nonetheless, it is reasonable to abstract from it in this paper. This is because export participation in an individual market (i.e., Germany in this paper) may plausibly have an insignificant effect on the overall evolution process of productivity.[7] Third, I model transaction-level exports rather than annual export sales directly. In the data set, firms usually have multiple export sales to a market within a year, presumably with different customers. This is crucial in characterizing demand learning. Export sales are noisy signals of the true demand, and the resolution of the uncertainty depends on how much information about the true demand has been accumulated. Firms with more orders per period learn faster because they observe more signals per period.[8] Thus, I explicitly model the persistence of orders to control for heterogeneous learning speeds.

In what follows, I describe the components of the model in detail. They include the firm's static decisions on production and pricing, the dynamic decision on export participation, as well as the evolution of the firm's belief about its foreign demand.

## 2.2 Static Decisions

The firm's static decision is to set prices for the domestic and foreign markets to maximizes profit in each period. In period $t$, firm $i$ faces a short-run marginal cost to produce a single product for the domestic and foreign markets. Within a firm and a period, the marginal cost is constant. But it is heterogeneous across firms and varies over time. Following Aw et al. (2011), I focus on

[6] If one is willing to assume that the firm learns about the global scope of demand (i.e., a firm-level demand component, which is common to all countries), then the identification and estimation methodology of this paper still works. Without this assumption, another alternative direction is to adopt a more complicated framework of correlated learning as in Marcoul and Weninger (2008).

[7] In a model with an AR(1) process of productivity evolution, I found that the productivity gain from exporting to Germany is insignificant.

[8] One potential concern is that firms may arbitrarily split orders into different transactions to learn faster. However, if this is the case, we would expect a larger number of orders for entrants into foreign markets than that of experienced firms. But in the data set the opposite is observed: experienced firms have 106 percent more transactions compared with entrants. In addition, shipping costs may prevent firms from arbitrarily splitting orders.

the heterogeneity that arises from firm differences in size (i.e., capital stock) and productivity. Specifically, the marginal cost in logarithm is:

$$\ln c_{it} = \gamma_0 + \gamma_k \ln k_{it} + \gamma_w \ln w_t - \omega_{it}, \tag{1}$$

where $k_{it}$ and $\omega_{it}$ are capital stock and physical productivity of firm $i$ in period $t$, respectively. Capital stock is observable to researchers and fixed in each period, but productivity is unobservable.[9] Time-specific input prices (i.e., wage rates) that are common to all firms are summarized in $w_t$. The marginal cost does not depend on the firm's output. This suggests that the firm's uncertainty about demand in foreign markets does not affect the static output decision in the domestic market.

In the domestic market, firm $i$ faces constant elasticity of demand:

$$q_{it}^D = \phi_t^D (p_{it}^D)^{\eta^D} e^{z_{it}^D}, \tag{2}$$

where $\eta^D < -1$ is the demand elasticity and $\phi_t^D$ is an aggregate demand shifter. $z_{it}^D = z_i^D + \nu_{it}^D$ is a domestic demand shock. I refer to $z_i^D$ as the domestic demand factor, which is known by the firm. It measures the time-invariant demand specific to firm $i$. $\nu_{it}^D$ is an i.i.d. draw from a mean-zero normal distribution.

After observing its marginal cost, domestic demand, and the aggregate shifter, the firm sets a price to maximize its profit in the domestic market. The first-order condition implies that the domestic market revenue is

$$R_{it}^D = \phi_t^D \left[\frac{\eta^D}{\eta^D + 1}\right]^{\eta^D+1} \left[e^{\gamma_0} k_{it}^{\gamma_k} w_t^{\gamma_w} e^{-\omega_{it} + \frac{1}{\eta^D+1} z_{it}^D}\right]^{\eta^D+1}. \tag{3}$$

[9] In reality, capital stock may evolve endogenously. More productive firms may choose to invest in capital and reduce their marginal cost. This suggests that firms with high productivity and capital are more likely to export. But in the empirical application, I consider a relatively short period and thus follow the literature (e.g., Aw et al., 2011) to treat capital stock with a degenerate transition in the dynamic model.

Define the domestic revenue-based productivity as[10]

$$\tilde{\omega}_{it} = \omega_{it} - \frac{1}{\eta^D + 1} z_{it}^D. \tag{4}$$

Therefore, the domestic revenue in logarithm can be written as:

$$\ln R_{it}^D = (\eta^D + 1) \ln \left( \frac{\eta^D}{\eta^D + 1} \right) + \ln \phi_t^D + (\eta^D + 1)(\gamma_0 + \gamma_k \ln k_{it} + \gamma_w \ln w_t - \tilde{\omega}_{it}). \tag{5}$$

That is, the domestic revenue is a function of the demand elasticity, domestic aggregate demand shifter, input prices, capital stock, and domestic revenue-based productivity. For simplicity, I refer to domestic revenue-based productivity as productivity unless otherwise mentioned in the remainder of the paper. It essentially contains both physical productivity and the domestic demand shock. I assume that it follows an exogenous first-order Markov process: $\tilde{\omega}_{it} = g(\tilde{\omega}_{it-1}) + \epsilon_{it}$, where $\epsilon_{it}$ is independently drawn from $N(0, \sigma_\epsilon)$.

In foreign market $j$, the firm's demand is heterogeneous in two aspects. First, firms are different in the customer base and size of the distribution network. This implies that the number of orders in a period varies across firms, conditional on exporting. This heterogeneity not only affects firm profitability, but also influences the speed of demand learning. I denote the number of orders that firm $i$ receives in period $t$ as $n_{it}^j$. I model it as a random draw from a truncated negative binomial distribution with mean $\lambda_{it}^j$ and dispersion parameter $\rho^j$.[11] Thus, the expected number of orders is $\lambda_{it}^j/(1-(1+\rho^j)^{-\lambda_{it}^j/\rho^j})$.[12] In the data set, firms with a large number of orders in the last year tend to secure many orders in this year as well. To capture such persistence, I allow the mean in the

[10] Foster et al. (2008) show that one can only recover a combination of the demand shock and physical productivity without output price data at the firm level. Aw et al. (2011) use domestic revenue-based productivity to control firms' heterogeneous export profitability in studying export participation and R&D decisions. I follow the practice in this study and assume that the firm observes both $\omega_{it}$ and $z_{it}^D$ when it makes static decisions.

[11] An alternative specification is to assume that the order process follows a Poisson distribution. However, given the skewness (3.5) of the number of orders in the data, the negative binomial distribution is more flexible than the Poisson distribution, although the key result of the paper using the Poisson distribution is quantitatively similar. In addition, the truncated distribution is used because I assume that if the firm exports, it will receive at least one order. When trade costs are random, as I assume in the model, one cannot tell whether a firm attempted but no order was received or it simply decided not to export because of a high trade cost shock.

[12] Under this setup, the probability of success in the standard negative binomial distribution is $1/(1+\rho^j)$.

negative binomial distribution to evolve over time:

$$\ln \lambda_{it}^j = \psi_0^j + \psi_n^j \ln(n_{it-1}^j + 1). \tag{6}$$

$\psi_n^j$ measures the persistence of the order process. Note that the firm's product price does not play any role here. Intuitively, if individual orders are from unique buyers, then lowering the price only influences the size of a given order rather than attracting more orders. This shares the same assumption in Arkolakis et al. (2017) and Berman et al. (forthcoming) that a change in the price shifts sales along the demand curve.

The second aspect of demand heterogeneity resides in the quantity demanded at a given price in an individual order. With a slight abuse of notation, I use $tn$ as the index of the firm's orders in period $t$. For each firm $i$, as in the domestic market, the quantity sold in an order in market $j$ is[13]

$$q_{itn}^j = \phi_t^j (p_{itn}^j)^{\eta^j} e^{z_{itn}^j}, \tag{7}$$

where $\eta^j < -1$ is the demand elasticity, $p_{itn}^j$ is the price, and $\phi_t^j$ is the aggregate demand shifter of market $j$. $z_{itn}^j = z_i^j + \nu_{itn}^j$ is the demand shock of order $tn$. As in the domestic market, $z_i^j$ measures the time-invariant demand specific to firm $i$. I refer to it as firm $i$'s demand factor in market $j$. $\nu_{itn}^j$ is an i.i.d. draw from a mean-zero normal distribution.

Conditional on exporting, the firm sets a price that maximizes the profit in each order.[14] The first-order condition and equation (4) imply that the revenue of order $tn$ (in logarithm) can be written as:

$$\ln r_{it}^j = (\eta^j+1) \ln \left( \frac{\eta^j}{\eta^j+1} \right) + \ln \phi_t^j + (\eta^j+1)(\gamma_0 + \gamma_k \ln k_{it} + \gamma_w \ln w_t - \tilde{\omega}_{it}) + \left( z_{itn}^j - \frac{\eta^j+1}{\eta^D+1} z_{it}^D \right). \tag{8}$$

I use lower case $r_{itn}^j$ to denote the revenue of an individual order, in order to distinguish it from total domestic revenue ($R_{it}^D$). Compared with its domestic counterpart in equation (5), $r_{it}^j$ has an

[13] This implies an analogy of the domestic demand function. That is, the demand of firm $i$ in period $t$ is the sum of quantity over all orders in this period: $E(q_{it}^j) = \lambda_{it}^j / (1 - (1+\rho^j)^{-\lambda_{it}^j/\rho^j}) \phi_t^j (p_{it}^j)^{\eta^j} E(e^{z_{itn}^j}) \equiv \bar{\phi}_{it}^j (p_{it}^j)^{\eta^j}$, where $\bar{\phi}_{it}^j$ is the product of $\lambda_{it}^j / (1 - (1+\rho^j)^{-\lambda_{it}^j/\rho^j})$, $E(e^{z_{itn}^j})$, and $\phi_t^j$.

[14] The pricing decision is static. It is in contrast to the modeling of active pricing and experimentation in Balvers and Cosimano (1990) and Bergemann and Välimäki (2000), where the firm's price influences the speed of learning.

additional determinant: $z^j_{itn} - \frac{\eta^j+1}{\eta^D+1} z^D_{it}$. Define

$$\zeta^j_{itn} = z^j_{itn} - \frac{\eta^j+1}{\eta^D+1} z^D_{it} = \overbrace{(z^j_i - \frac{\eta^j+1}{\eta^D+1} z^D_i)}^{\xi^j_i} + \overbrace{(\nu^j_{itn} - \frac{\eta^j+1}{\eta^D+1} \nu^D_{it})}^{u^j_{itn}} \equiv \xi^j_i + u^j_{itn} \tag{9}$$

as the difference between the demand shocks in the domestic market and export market $j$ adjusted by the demand elasticities. That is, $\xi^j_i$ is the adjusted difference between the demand factors in the two markets and $u^j_{itn}$ is the adjusted difference between the idiosyncratic shocks. As such, $\xi^j_i$ is a constant and $u^j_{itn}$ has a normal distribution (with standard deviation $\sigma_u$).

The demand and cost structure implies a simple link between the profit of an individual order and its revenue: $\pi^j_{itn} = -\frac{1}{\eta^j} r^j_{itn}$. Consequently, the expected total export profit in market $j$ is the sum of profit generated by all orders in period $t$:[15]

$$\begin{aligned} E(\Pi^j_{it}) &= E_{n^j_{it}, \zeta^j_{itn}} \Big( \sum_{tn=1}^{n^j_{it}} \pi^j_{itn} \Big) \\ &= \frac{\lambda^j_{it} \phi^j_t}{1 - (1+\rho^j)^{-\lambda^j_{it}/\rho^j}} \; \frac{-1}{1+\eta^j} \Big[ \frac{\eta^j}{1+\eta^j} \Big]^{\eta^j} \Big[ e^{\gamma_0} k^{\gamma_k}_{it} w^{\gamma_w}_t e^{-\tilde{\omega}_{it}} \Big]^{1+\eta^j} e^{\sigma^2_u/2} E(e^{\xi^j_i}). \end{aligned} \tag{10}$$

This equation shows how domestic revenue-based productivity ($\tilde{\omega}_{it}$) and the demand difference ($\xi^j_i$) together affect the expected export profit. $\tilde{\omega}_{it}$ influences the expected export profit by giving the firm a reference profitability based on the firm's performance in the domestic market. $\xi^j_i$ adjusts the reference toward expected foreign profitability.

The firm faces uncertainty about the foreign demand factor $z^j_i$. Because the domestic demand factor $z^D_i$ is known by the firm, the uncertainty about $z^j_i$ is equivalent to the uncertainty about $\xi^j_i$. Thus, I model the firm's demand learning as a process of learning about $\xi^j_i$. As shown in equation (10), this enables me to utilize domestic revenue-based productivity directly in modeling the expected export profit.

As a result, the firm's belief about $\xi^j_i$ *directly* influences the firm's expected export profit and consequently export participation in market $j$, given $\tilde{\omega}_{it}$ and other variables. The firm can learn

[15] The last equation holds because $E(e^{\zeta^j_{itn}}) = e^{\sigma^2_u/2} E(e^{\xi^j_i})$ and the expected number of orders is $\lambda^j_{it}/(1-(1+\rho^j)^{-\lambda^j_{it}/\rho^j})$.

about the true value of $\xi_i^j$ from its own exporting experience. That is, the firm observes the revenue of each order *after* exporting, and the comparison between it and the domestic revenue serves as a noisy signal about $\xi_i^j$. In particular, using equation (5) and (8), it is straightforward to show that the adjusted difference between sales in the domestic market and an export transaction reflects a realization of $\zeta_{itn}^j$:

$$\zeta_{itn}^j = \ln r_{itn}^j - \left(\frac{1+\eta^j}{1+\eta^D}\right) \ln R_{it}^D - \ln \Phi_t^j, \tag{11}$$

where $\Phi_t^j = \phi_t^j [\phi_t^D]^{-\frac{1+\eta^j}{1+\eta^D}} \left(\frac{\eta^j}{1+\eta^j} \frac{1+\eta^D}{\eta^D}\right)^{\eta^j+1}$. That is, $(r_{itn}^j, R_{it}^D)$ contains information about $\zeta_{itn}^j$. Because $\zeta_{itn}^j$ is a signal of $\xi_i^j$, the firm is able to update its belief regarding $\xi_i^j$. The next subsection explains this learning process in detail.

### 2.3 Demand Uncertainty and Bayesian Learning by Exporting

This subsection characterizes Bayesian learning about foreign demand in a single market $j$. Since I assume the learning processes in different markets are independent, I omit the superscript $j$ to simplify the notation.

The objective of the learning process is $\xi_i$.[16] The firm knows the distribution of $\xi$ in the entire industry. This serves as a prior belief about its own $\xi_i$. In general, observable characteristics may influence firms' prior beliefs. For example, firms may believe that demand is higher in markets with larger populations. Formally, the firm holds a prior belief $\xi_i \sim N(m_{i0}, \sigma_{i0})$ at the beginning of the initial period.[17] Specifically, $m_{i0} = h_m(x_i; \beta_m)$ is the prior expectation of $\xi_i$ and $\sigma_{i0} = h_\sigma(x_i; \beta_\sigma)$ captures the initial uncertainty. $\beta_m$ and $\beta_\sigma$ are the associated parameters, and $x_i$ is a set of firm and market characteristics, such as firm age, ownership, size, and the population of the market.

The firm can learn about the true value of $\xi_i$ by observing $\zeta_{itn}$ from each of its orders, conditional on exporting. Note that $\zeta_{itn} = \xi_i + u_{itn}$, but $\xi_i$ and the unexpected idiosyncratic shock $u_{itn}$ are not separately observed. For example, the firm cannot tell whether a big export sale is due to persistent high demand or temporary luck. That is, the value of $\xi_i$ is not immediately revealed because of the noise $u_{itn}$. However, the firm knows the distribution of the noise: $u_{itn} \sim N(0, \sigma_u)$. This enables

[16] Another way to model demand uncertainty is to assume $\lambda_{it}$ is unknown and to be learned by the firm. However, the variance of the number of orders is large and suggests a learning speed that is too low to match the data.

[17] Empirically, the "initial period" means the first period when a firm appears in the data set. It is not necessarily the period of its first export, because the firm's domestic production (thus their existence) is observed even if it does not export.

the firm to update its belief according to Bayes' rule after observing a series of signals.[18] The standard deviation $\sigma_u$ measures the informativeness of a single signal and thus influences how fast the uncertainty resolves. In the extreme case where $\sigma_u = 0$, $\xi_i$ can be accurately revealed after just one order. However, if $\sigma_u$ is large, the firm needs more signals to achieve a given level of accuracy. As such, given $\sigma_u$, firms with more orders per period can uncover their underlying foreign demand faster. The implication of heterogeneous learning speeds is taken into account by order process (6).

More specifically, let demand belief at the beginning of period $t$ be $\xi_i \sim N(m_{it-1}, \sigma_{it-1})$. If the firm decides not to export in period $t$, then the firm will not observe any new signals and there is no change in demand belief at the end of period $t$. However, if the firm exports and $n_{it}$ orders are received in period $t$, then the firm will update its demand belief with $n_{it}$ new signals $\{\zeta_{i1}, \zeta_{i2}, \ldots, \zeta_{in_{it}}\}$. Consequently, the firm's posterior demand belief at the end of period $t$ is given by $\xi_i \sim N(m_{it}, \sigma_{it})$, where[19]

$$m_{it} = \begin{cases} \dfrac{\sigma_u^2 m_{it-1} + n_{it}\sigma_{it-1}^2 \tilde{\zeta}_{it}}{\sigma_u^2 + n_{it}\sigma_{it-1}^2}, & \text{if exported in period } t \\ m_{it-1}, & \text{otherwise} \end{cases} \tag{12}$$

and

$$\sigma_{it}^2 = \begin{cases} \dfrac{\sigma_{it-1}^2 \sigma_u^2}{\sigma_u^2 + n_{it}\sigma_{it-1}^2}, & \text{if exported in period } t \\ \sigma_{it-1}^2, & \text{otherwise} \end{cases} \tag{13}$$

and

$$\tilde{\zeta}_{it} = \frac{1}{n_{it}} \sum_{tn=1}^{n_{it}} \zeta_{itn}.$$

Alternatively, the above equations can be written in terms of the firm's initial demand belief and the entire history of signals received until period $t$: $\{\zeta_{i1}, \zeta_{i2}, \ldots, \zeta_{iN_{it}}\}$ where $N_{it} = \sum_{\tau=1}^{t} n_{i\tau}$. That is,

$$m_{it} = \frac{\sigma_u^2 m_{i0} + N_{it}\sigma_{i0}^2 \overline{\zeta}_{it}}{\sigma_u^2 + N_{it}\sigma_{i0}^2}, \tag{14}$$

[18] The firm may update its belief after each transaction. But this is equivalent to the update at the end of the period because the firm only makes the export decision at the beginning of each period.

[19] See DeGroot (2005), chapter 9.

and

$$\sigma_{it}^2 = \frac{\sigma_{i0}^2 \sigma_u^2}{\sigma_u^2 + N_{it}\sigma_{i0}^2}, \tag{15}$$

and

$$\overline{\zeta}_{it} = \frac{1}{N_{it}} \sum_{tn=1}^{N_{it}} \zeta_{itn}. \tag{16}$$

In each period $t$, the firm's belief about $\xi_i$ is characterized by two variables: the mean ($m_{it}$) and the standard deviation ($\sigma_{it}$). The mean represents the expectation of $\xi_i$ and may fluctuate over time depending on the entire history of signals that the firm has received. The standard deviation measures the magnitude of uncertainty and is strictly decreasing as the firm accumulates more signals.

### 2.4 Dynamic Decision – Export Participation with Demand Learning

This subsection characterizes a forward-looking firm's export participation when it learns about its demand in market $j$. Again, the superscript $j$ is omitted to simplify the notation. The terms "export" and "not to export" refer to the firm's decision about export participation in market $j$.

In this model, the export decision and demand learning process are endogenously related. The firm's export decision depends on its current belief about its foreign demand. If the firm decides to export, then it expects to update its demand belief in the next period according to the signals received from exporting. Moreover, the sunk cost of exporting makes the export decision dynamic. The literature has documented the significant impact of sunk cost on export participation (e.g., Roberts and Tybout, 1997; Das et al., 2007). For example, large sunk cost may prevent the firm from entering the foreign market immediately after a positive productivity shock. Thus, both the learning process and sunk export cost contribute to the persistence of export participation. Ignoring the sunk cost could potentially bias the role of demand learning.

To model the sunk cost explicitly, I allow the export costs to depend on not only current export status, but also lagged export status. Intuitively, new exporters may need to pay higher costs (referred to as sunk costs) to start exporting compared with continuing exporters who have established distribution networks and only pay continuation costs (referred to as fixed costs).[20]

[20] These trade costs are country-specific. In principle, both country-specific and global trade costs may exist. However, their relative sizes may vary across countries and industries. Moxnes (2010) uses a panel of Norwegian manufacturers and argues that global costs are about one-third of country-specific costs. However, McCallum (2013)

That is,

$$C(e_{it-1}, e_{it}) = e_{it-1}e_{it}c^f + (1 - e_{it-1})e_{it}c^s - \delta\epsilon(e_{it}), \tag{17}$$

where $e_{it-1}$ and $e_{it}$ are binary variables that indicate export status in periods $t-1$ and $t$, respectively. $c^f$ and $c^s$ are the constant terms of the fixed cost and sunk cost, respectively. $\epsilon(e_{it})$ with $e_{it} = \{0, 1\}$ captures idiosyncratic shocks to the export costs. It is an independent draw from the standard Type-I extreme distribution with a cumulative distribution function $G(\epsilon) = -\exp(-\exp(\epsilon))$. $\delta$ measures the scale of the shock.

According to equation (10), given the current belief $N(m_{it-1}, \sigma_{it-1})$, the expected export profit in period $t$ (before considering export costs) is

$$E\Big[\Pi(s_{it})\Big] = \frac{\lambda_{it}\tilde{\phi}_{it}}{1 - (1+\rho)^{-\lambda_{it}/\rho}} \exp(m_{it-1} + \sigma_{it-1}^2/2 + \sigma_u^2/2 - (\eta+1)\tilde{\omega}_{it}), \tag{18}$$

where $\tilde{\phi}_{it} = \frac{-\phi_t}{1+\eta}\left[\frac{\eta}{1+\eta}\right]^{\eta} e^{\gamma_0(1+\eta)} w_t^{\gamma_w(1+\eta)} k_{it}^{\gamma_k(1+\eta)}$. That is, the expected export profit $E[\Pi(s_{it})]$ depends on the expected number of orders, productivity, current demand belief (characterized by $m_{it-1}$ and $\sigma_{it-1}$), and $\tilde{\phi}_{it}$. Importantly, the expected export profit is increasing in $m_{it-1}$ and $\sigma_{it-1}$. Firms with higher demand expectation or greater uncertainty have higher expected export profit and are more likely to export, holding other variables fixed.

To simplify the notation, I summarize the set of state variables as $(s_{it}, e_{it-1}) \equiv (\tilde{\phi}_{it}, \lambda_{it}, \tilde{\omega}_{it}, m_{it-1}, \sigma_{it-1}, e_{it-1})$ and define the expected net profit of exporting as

$$u(s_{it}, e_{it-1}) = E[\Pi(s_{it})] - e_{it-1}c^f - (1 - e_{it-1})c^s. \tag{19}$$

The timing of the entire model is summarized as follows. At the beginning of period $t$, the firm observes state $(s_{it}, e_{it-1})$ and export costs $C(e_{it-1}, e_{it})$. Then, the firm decides whether or not to export. If the firm decides to export, then it pays the export costs and earns export profit as in equation (18). At the end of this period, the firm updates its belief using new signals

finds a much lower relative size of global costs using U.S. manufacturing firms: the global costs are as small as 1 percent of country-specific costs. In the context of exports to Germany in this paper, I find that the lagged export status to European countries other than Germany does not have significant prediction power of the current export status to Germany. This suggests that, if it exists, the global sunk cost of entering the European Union is much smaller than the country-specific cost. Thus, I abstract it from the model.

$\{\zeta_{i1}, \zeta_{i2}, \ldots, \zeta_{in_{it}}\}$. The posterior belief is $N(m_{it}, \sigma_{it})$, according to equations (12) and (13). If the firm decides not to export, then there is no export profit, and there is no change in demand belief at the end of this period. Finally, period $t+1$ begins and all other state variables are updated.

Given the timing, I model the firm's dynamic export participation using a Bellman equation. I denote the expected value function at the beginning of period $t$ as $V(s_{it}, e_{it-1})$. The firm chooses to export if the expected value of exporting net of export costs is greater than the expected value of not exporting. Thus, the Bellman equation is given by

$$\begin{aligned} &V(s_{it}, e_{it-1}) \\ &= E_{\epsilon(0),\epsilon(1)} \max \begin{cases} \delta\epsilon(0) + \beta E[V(s_{it+1}, 0)|s_{it}, e_{it-1}], & \text{if} \quad e_{it} = 0 \\ u(s_{it}, e_{it-1}) + \delta\epsilon(1) + \beta E[V(s_{it+1}, 1)|s_{it}, e_{it-1}], & \text{if} \quad e_{it} = 1 \end{cases} \end{aligned} \tag{20}$$

where $\beta$ is the discount rate, and

$$E[V(s_{it+1}, e_{it})|s_{it}, e_{it-1}] = \int V(s_{it+1}, e_{it}) dF_{\tilde{\omega}}(\tilde{\omega}_{it+1}|\tilde{\omega}_{it}) dF_b(m_{it}, \sigma_{it}, \lambda_{it+1}|s_{it}, e_{it-1}, e_{it}). \tag{21}$$

$F_{\tilde{\omega}}(\tilde{\omega}_{it+1}|\tilde{\omega}_{it})$ and $F_b(m_{it}, \sigma_{it}, \lambda_{it+1}|s_{it}, e_{it-1}, e_{it})$ are the transition probabilities of the four key state variables. I specify these transition probabilities as follows.

First, $F_{\tilde{\omega}}(\tilde{\omega}_{it+1}|\tilde{\omega}_{it})$ is the distribution of $\tilde{\omega}_{it+1}$ conditional on $\tilde{\omega}_{it}$. The evolution of $\tilde{\omega}$ is $\tilde{\omega}_{it+1} = g(\tilde{\omega}_{it}) + \epsilon_{it+1}$. Thus,

$$F_{\tilde{\omega}}(\tilde{\omega}_{it+1}|\tilde{\omega}_{it}) = N(g(\tilde{\omega}_{it}), \sigma_\epsilon). \tag{22}$$

Second, $F_b(m_{it}, \sigma_{it}, \lambda_{it+1}|s_{it}, e_{it-1}, e_{it})$ is the joint distribution of demand belief and $\lambda_{it+1}$ at the beginning of period $t+1$, given current state $s_{it}$ and export status $(e_{it-1}, e_{it})$. It depends on the export participation decision. If the firm does not export in period $t$, then the posterior demand belief at the beginning of period $t+1$ remains the same. Therefore, the transition probability is

$$F_b(m_{it} = m_{it-1}, \sigma_{it} = \sigma_{it-1}, \lambda_{it+1}|s_{it}, e_{it-1}, e_{it} = 0) = 1,$$

where $\lambda_{it+1} = \exp(\psi_0 + \psi_n \ln(n_{it} + 1))$. By contrast, if the firm exports, and suppose it receives $n_{it}$

new signals $\{\zeta_{i1}, \zeta_{i2}, \ldots, \zeta_{in_{it}}\}$ in this period, then the updated demand belief is $N(m_{it}, \sigma_{it})$, where

$$m_{it} = m_{it-1} + n_{it}\frac{\sigma_{it}^2}{\sigma_u^2}\Big(\frac{1}{n_{it}}\sum_{tn=1}^{n_{it}}(\xi_i + u_{itn}) - m_{it-1}\Big) \tag{23}$$

and

$$\sigma_{it}^2 = \frac{\sigma_{it-1}^2\sigma_u^2}{\sigma_u^2 + n_{it}\sigma_{it-1}^2}. \tag{24}$$

Because $m_{it-1}$ and $\sigma_{it-1}$ are known by the firm as a part of the state variable $s_{it}$ in period $t$, the transition depends on the distributions of $(u_{itn}, \xi_i, n_{it})$. The distribution of $u_{itn}$ is known as $N(0, \sigma_u)$, and $\xi_i$ is believed to be distributed as $N(m_{it-1}, \sigma_{it-1})$. Thus, *conditional on* $n_{it}$, the distribution of $m_{it}$ is[21]

$$m_{it} \sim N\Big(m_{it-1}, n_{it}\frac{\sigma_{it}^2}{\sigma_u^2}\sqrt{\sigma_u^2/n_{it} + \sigma_{it-1}^2}\Big) \equiv F_m(m_{it}|n_{it}, s_{it}, e_{it} = 1), \tag{25}$$

and the distribution of $\sigma_{it}$ is

$$F_\sigma\left(\sigma_{it} = \sqrt{\frac{\sigma_{it-1}^2\sigma_u^2}{\sigma_u^2 + n_{it}\sigma_{it-1}^2}}\middle| n_{it}, s_{it}, e_{it} = 1\right) = 1. \tag{26}$$

Because $n_{it}$ is drawn from a truncated negative binomial distribution with known parameters $\lambda_{it}$ and $\rho$, the probability to receive $n_{it}$ orders in period $t$ is

$$F_n(n_{it}|s_{it}, e_{it-1}) = \frac{\Gamma(n_{it} + \lambda_{it}/\rho)}{n_{it}!\Gamma(\lambda_{it}/\rho)}\left(\frac{\rho}{\rho+1}\right)^{n_{it}}\left(\frac{1}{\rho+1}\right)^{\frac{\lambda_{it}}{\rho}}\frac{1}{1-(1+\rho)^{-\lambda_{it}/\rho}}, \tag{27}$$

where $\Gamma(\cdot)$ is the gamma function.

Therefore, the joint transition probability of $(m_{it}, \sigma_{it}, \lambda_{it+1})$ is given by

$$\begin{aligned} &F_b(m_{it}, \sigma_{it}, \lambda_{it+1}|s_{it}, e_{it-1}, e_{it} = 1) \\ &= F_m(m_{it}|n_{it}, s_{it}, e_{it} = 1)F_\sigma(\sigma_{it}|n_{it}, s_{it}, e_{it} = 1)F_n(n_{it} = \psi^{-1}(\lambda_{it+1})|s_{it}, e_{it-1}), \end{aligned} \tag{28}$$

where $\psi^{-1}(\cdot)$ is the inverse function of order process (6).

In sum, these transition probabilities are driven by the evolutions of productivity and demand belief. They enter the Bellman equation as a part of equation (21), and they affect the firm's

[21] According to order process (6), knowledge of $n_{it}$ and $\lambda_{it+1}$ is equivalent.

optimal decision on export participation by influencing the firm's expectation of the future value of exporting. The next section describes the strategy of identifying the effects of the two processes in export dynamics and estimating the structural model.

# 3 Identification and Estimation Strategy

Both productivity and demand belief are heterogeneous across firms and evolve over time, but neither of them is observable to researchers. This places a challenge to separate the roles of the two forces. My identification strategy relies on data from two sources. The first data source provides firm-level production information, which includes domestic revenue, labor and material expenditures, capital stock, and other firm characteristics. The second data source contains firms' transaction-level exports, including the export destination and revenue of each transaction.

I use the firm-level production data to estimate a time-varying measure of domestic revenue-based productivity for each firm. Due to the lack of firm-level output prices in the domestic market, the productivity measure based on domestic revenue is a combination of both the physical productivity and domestic demand shock (Foster et al., 2008). To accommodate this, demand belief is defined on the difference between foreign demand and domestic demand. As a result, export profit (thus, export participation) depends on the firm's domestic revenue-based productivity and history-dependent demand belief, as described in Section 2.

I use transaction-level exports to construct demand signals and update firms' demand beliefs over time. Specifically, the history of export transaction revenue is used to update the expectation of demand belief. The history of the number of transactions is used to determine the resolution of uncertainty. The propensity of exporting depends on the expectation and uncertainty of demand belief in different ways. The propensity to export is increasing in the expectation, which fluctuates over time depending on the observed sizes of individual transactions. However, because uncertainty is resolved over time as the firm gains more experience, the propensity to export is decreasing in the accumulated number of transactions.

A classical model with only productivity heterogeneity predicts that more productive firms export. In this model with two-dimensional heterogeneity, firms that face greater demand uncertainty may also export even if their productivity is not high. Thus, with productivity and demand beliefs

recovered, the cross-sectional and time-series variation of export participation helps to identify the role of each driving force. In the estimation, I first estimate time-varying productivity, the history of demand signals, and the associated parameters. Then, I estimate the dynamic model of export participation.

### 3.1 Estimation of Static Parameters, Productivity, and Demand Signals

For the first step, I estimate the static parameters: demand elasticities in the domestic and foreign markets and marginal cost parameters. Specifically, I use the data on firm-level production and domestic revenue to estimate the marginal cost function and domestic revenue-based productivity. Then I estimate the demand elasticities from the relationship between firms' total variable costs and market-level exports. Finally, I use individual transaction-level exports to recover demand signals. The implementation of this strategy is described as follows.

The domestic revenue function, equation (5), implies an empirical equation of observed domestic revenue

$$\ln R_{it}^D = (\eta^D + 1) \ln \left( \frac{\eta^D}{\eta^D + 1} \right) + \ln \phi_t^D + (\eta^D + 1)(\gamma_0 + \gamma_k \ln k_{it} + \gamma_w \ln w_t - \tilde{\omega}_{it}) + \upsilon_{it}, \tag{29}$$

where $\upsilon_{it}$ is a measurement error. Productivity $\tilde{\omega}_{it}$ is observed by the firm and is correlated with capital stock. But it is unobserved by researchers. Thus, following Levinsohn and Petrin (2003), I write $\tilde{\omega}_{it}$ as a function of material inputs, conditional on capital stock. That is, $\tilde{\omega}_{it} = \tilde{\omega}(k_{it}, m_{it})$.[22] Then, I combine the demand elasticity terms into an intercept $\tilde{\gamma}_0^D$ and use a set of time dummies, $\tilde{\Phi}_t^D = \phi_t^D w_t^{(1+\eta^D)\gamma_w}$, to capture the domestic industrial aggregate shifter and input prices, which are common to all firms. Thus, the above equation can be written as:

$$\ln R_{it}^D = \tilde{\gamma}_0^D + \sum_{t=1}^{T-1} \gamma_t \tilde{\Phi}_t^D + f(k_{it}, m_{it}) + \upsilon_{it}, \tag{30}$$

where $f(k_{it}, m_{it}) = (1 + \eta^D)(\gamma_k \ln k_{it} - \tilde{\omega}(k_{it}, m_{it}))$ depends on capital stock and material inputs. I

[22] Online Appendix A presents and discusses measures of productivity estimated from different methodologies, including Olley and Pakes (1996), Levinsohn and Petrin (2003), and Wooldridge (2009), using investment, material inputs, or domestic revenue-weighted material inputs in the control function of productivity. Overall, the cost parameters associated with different estimation methods are quantitatively similar. More importantly, these productivity measures are highly correlated and demonstrate a robust reduced-form relationship to firm export participation.

parameterize it as a function of $k_{it}$ and $m_{it}$ using a set of Legendre polynomials of degree three. The error term $\upsilon_{it}$ is uncorrelated with the right-hand-side variables. I use an ordinary least squares regression to obtain the estimates. An output from the regression is the fitted value of function $f$:

$$\hat{f}_{it} = (1 + \eta^D)(\gamma_k \ln k_{it} - \tilde{\omega}_{it}). \tag{31}$$

I specify the first-order Markov process of productivity evolution as an AR(1) process:

$$\tilde{\omega}_{it} = g_0 + g_1 \tilde{\omega}_{it-1} + \epsilon_{it}. \tag{32}$$

Substituting $\tilde{\omega}_{it} = -\frac{1}{\eta^D+1}\hat{f}_{it} + \gamma_k \ln k_{it}$ into the above evolution process yields

$$\hat{f}_{it} = -(\eta^D + 1)g_0 + g_1 \hat{f}_{it-1} + (\eta^D + 1)\gamma_k \ln k_{it} - g_1(\eta^D + 1)\gamma_k \ln k_{it-1} - (\eta^D + 1)\epsilon_{it}.$$

The error term is uncorrelated with other right-hand-side variables. I estimate the equation by a nonlinear least squares regression. The parameters estimated in this equation are $g_0^* = (\eta^D + 1)g_0$, $g_1$, and $\gamma_k^* = (\eta^D + 1)\gamma_k$. Note that $\eta^D$, $g_0$, and $\gamma_k$ are not separately identified. However, if $\eta^D$ is known, then $g_0 = \frac{g_0^*}{\eta^D+1}$ and $\gamma_k = \frac{\gamma_k^*}{\eta^D+1}$ are immediately recovered. Importantly, from equation (31), I can recover productivity as $\tilde{\omega}_{it} = -\frac{1}{\eta^D+1}\hat{f}_{it} + \gamma_k \ln k_{it}$, once $\eta^D$ is estimated.

To estimate $\eta^D$, I follow Aw et al. (2011) to utilize the relationship between total variable costs and total revenue in different markets. Specifically, the first-order conditions for profit maximization of the domestic and foreign markets suggest:

$$TVC_{it} = \sum_{j=D,1,2,\ldots,J} q_{it}^j c_{it} = \sum_{j=D,1,2,\ldots,J} \left(1 + \frac{1}{\eta^j}\right) q_{it}^j p_{it}^j = \sum_{j=D,1,2,\ldots,J} \left(1 + \frac{1}{\eta^j}\right) R_{it}^j,$$

where $TVC_{it}$ is the total variable costs, $q_{it}^j$ is the total quantity sold to market $j$ by firm $i$ in period $t$, and $R_{it}^j$ is the corresponding total revenue. Therefore, I use the following empirical equation to estimate the demand elasticities:

$$TVC_{it} = \sum_{j=D,1,2,\ldots,J} (1 + \frac{1}{\eta^j}) R_{it}^j + \upsilon_{it}^c, \tag{33}$$

where $\upsilon_{it}^{c}$ is a measurement error.

With the estimated demand elasticities, I recover the aggregate demand shifter $\phi_t^j$ in the foreign demand function (7) by estimating equation (11) via a regression with fixed effects:

$$\ln r_{itn}^j - \left(\frac{1+\hat{\eta}^j}{1+\hat{\eta}^D}\right) \ln R_{it}^D = \ln \Phi_t^j + \xi_i^j + u_{itn}^j, \tag{34}$$

The estimate $\hat{\Phi}_t^j$ reveals the aggregate demand shifter $\hat{\phi}_t^j$, because all the other constants have been estimated. The regression also provides an estimate of the standard deviation of the noise as $\hat{\sigma}_u^j$.

After obtaining these estimates, I can recover the demand signal from an individual export transaction $tn$ of firm $i$ in period $t$ as:

$$\zeta_{itn}^j = \ln r_{itn}^j - \left(\frac{1+\hat{\eta}^j}{1+\hat{\eta}^D}\right) \ln R_{it}^D - \ln \hat{\Phi}_t^j. \tag{35}$$

That is, the comparison of the revenue of an individual export transaction and the elasticity-adjusted domestic revenue serves as a signal of the demand difference, after controlling for the aggregate demand shifter. Then, I use the history of demand signals to update the firms' market-specific demand beliefs according to Bayes' rule, as specified in equations (14) and (15).

To sum up, the static stage estimates time-varying productivity and demand signals for each firm. In effect, I use domestic revenue data to estimate productivity, which is one source of the underlying profit heterogeneity in the export market. Another source of the profit heterogeneity is foreign demand, about which firms hold beliefs. I use transaction-level exports to recover the history of demand signals and use them to update firms' beliefs.

### 3.2 Estimation of Dynamic Parameters

The set of dynamic parameters includes the parameters in the initial belief functions, the trade cost distribution, and the order process. I estimate them via the maximum likelihood method. The likelihood is constructed from the data on export participation, the number of export transactions, as well as dynamic state variables including productivity and demand signals recovered in the static estimation stage. To simplify the notation, I consider a single market $j$ and omit the market superscript.

The full likelihood consists of two partial likelihoods. The first one is for the number of transactions $n_{it}$. The parameters include those in the evolution process of $\lambda_{it}$ and the dispersion parameter of the negative binomial distribution. I summarize them in $\theta_1 = (\psi_0, \psi_n, \rho)$. The first partial likelihood is determined by the truncated negative binomial probability:

$$\ell^1(n_{it};\theta_1) = \frac{\Gamma(n_{it}+\lambda_{it}/\rho)}{n_{it}!\Gamma(\lambda_{it}/\rho)}\left(\frac{\rho}{\rho+1}\right)^{n_{it}}\left(\frac{1}{\rho+1}\right)^{\frac{\lambda_{it}}{\rho}}\frac{1}{1-(1+\rho)^{-\lambda_{it}/\rho}},$$

where $\lambda_{it}$ is specified in equation (6).

The second partial likelihood is for the decision on export participation $e_{it}$ conditional on ($s_{it}$, $e_{it-1}$). The associated parameters are the parameters of the initial beliefs and the trade cost distribution. I summarize them as $\theta_2 = (\beta_m, \beta_\sigma, c^f, c^s, \delta)$. The second partial likelihood is

$$\ell^2(e_{it}|s_{it}, e_{it-1};\theta_1,\theta_2) = e_{it}\Pr(e_{it}=1|s_{it}, e_{it-1};\theta_1,\theta_2) + (1-e_{it})\Pr(e_{it}=0|s_{it}, e_{it-1};\theta_1,\theta_2),$$

where $\Pr(e_{it}=1|s_{it}, e_{it-1};\theta_1,\theta_2)$ is the conditional probability of exporting, which depends on both $\theta_1$ and $\theta_2$. Given the standard Type-I extreme value distribution of the trade cost shocks, the conditional probability of exporting is

$$\begin{aligned}
&\Pr(e_{it}=1|s_{it}, e_{it-1};\theta_1,\theta_2)\\
&= \Pr\Big(u(s_{it}, e_{it-1}) + \delta E[V(s_{it+1},1)|s_{it}, e_{it-1}] - \beta E[V(s_{it+1},0)|s_{it}, e_{it-1}] > \epsilon(0)-\epsilon(1)\Big) \qquad (36)\\
&= \frac{\exp(u(s_{it}, e_{it-1}) + \beta E[V(s_{it+1},1)|s_{it}, e_{it-1}])}{\exp(u(s_{it}, e_{it-1}) + \beta E[V(s_{it+1},1)|s_{it}, e_{it-1}]) + \exp(\beta E[V(s_{it+1},0)|s_{it}, e_{it-1}]))},
\end{aligned}$$

where $u(s_{it}, e_{it-1})$ is the expected net export profit for current period $t$ given by equation (19), and $E[V(s_{it+1}, e_{it})|s_{it}, e_{it-1}]$ is the expected firm value at the beginning of period $t+1$ as described in the Bellman equation (20). Both the demand belief and productivity affect the probability of exporting through $u(s_{it}, e_{it-1})$ and $E[V(s_{it+1}, e_{it})|s_{it}, e_{it-1}]$.

The full likelihood function is the product of the two partial likelihoods:

$$\ell^f(e_{it}, n_{it}|s_{it}, e_{it-1};\theta_1,\theta_2) = \ell^1(n_{it};\theta_1)\ell^2(e_{it}|s_{it}, e_{it-1};\theta_1,\theta_2).$$

The full set of dynamic parameters is denoted by $\theta = (\theta_1, \theta_2)$. I estimate $\theta$ by maximizing the

full likelihood in logarithm:[23]

$$\hat{\theta} = \arg\max_{\theta} \sum_{i,t} \ln(\ell^f(e_{it}, n_{it}|s_{it}, e_{it-1}; \theta_1, \theta_2)). \tag{37}$$

In principle, the maximum likelihood estimation procedure involves repeatedly evaluating the full likelihood function to search the parameter space. Each evaluation of the full likelihood function requires solving the unknown value function implicitly defined by the functional Bellman equation (20). This is commonly known as the nested fixed point algorithm, following Rust (1987). However, in this application, the high dimension of state variables and number of structural parameters reduce the feasibility to compute the value function efficiently. This inhibits the use of the nested fixed point algorithm. More importantly, Dubé et al. (2012) demonstrate that the typical implementation of the nested fixed point algorithm may lead to incorrect parameter estimates. To address these challenges, I use the methodology of mathematical program with equilibrium constraints. This methodology was established by Su and Judd (2012) and Dubé et al. (2012). It was further implemented in Barwick and Pathak (2015). The idea is to approximate the unknown value function in the Bellman equation using sieves with parametric basis functions (Chen, 2007).[24] As a result, the Bellman equation can be cast as a model constraint in the estimation procedure that has to be satisfied at the parameter estimates. As such, it does not need to solve the Bellman equation repeatedly in an inner loop of the estimation procedure. Thus, it avoids the associated numerical issues in the nested fixed point algorithm, as demonstrated by Dubé et al. (2012).[25]

# 4 Data

## 4.1 Data Sources

I use data on firm-level production and transaction-level exports to Germany in the Chinese ceramics and glass industry to estimate the structural model. The data is drawn from two sources. The first source is the Chinese Annual Survey of Industrial Firms, which is collected by the Chinese National Bureau of Statistics. This database covers 2000 to 2006. It provides detailed, annual firm-level

[23] The details are described in Online Appendix C.

[24] The validity and properties of the implementation using sieve basis functions are discussed in Barwick and Pathak (2015).

[25] The implementation in my application is described in Online Appendix D.

production information on all medium-size and large manufacturing firms that had total annual sales greater than US$600,000. The primary variables include the firm-level domestic revenue, capital stock, intermediate input expenditure, and wage bill. I use them to estimate a firm-time-specific productivity measure and the marginal production cost function as described in Section 3.

The second source is the Chinese Customs Transactions data. It includes all export transactions of Chinese firms from 2000 to 2006. It is common for firms to have multiple transactions to an individual export market within a year. Across firms, the median number of transactions to Germany in a year is 4 and the standard deviation is 6.1. As such, it is important to model the number of orders explicitly to control for the difference in learning speeds across firms. Compared with the typical firm-level trade data sets used in the literature, this data set offers richer information. It contains the history of all export transactions during the data period. Each transaction contains transaction revenue, quantity, eight-digit Harmonized System code, destination market, shipment month, and firm identification number. The transaction-level revenue enables me to construct the demand signals, and the number of transactions measures firms' export experience. Both of them vary across firms and over time. Such detailed information is crucial to model and estimate the demand learning process.

There are 1,101 firms in the ceramics and glass industry that can be identified from both data sources.[26] The estimation is based on the data on these firms and their exports to Germany. There are several reasons for this. First, most firms in this industry have a concentrated product line. Their main products are sanitation, special, and daily-use ceramics and glass, and the firms do not operate in multiple industries.[27] Second, the number of firms is large, and this makes it appropriate to abstract from firms' strategic interactions. Indeed, the maximum market share in the export market among these firms is 4.3 percent. Third, these firms are likely to face demand uncertainty in Germany. This may be due to the geographic distance and potentially different consumer tastes. In particular, Germany is the fifth popular (rather than the most popular) export market by total

[26]This implies that these are non-trivial firms. They are identified from both data sources only if they satisfy two conditions. First, they had international engagement (to be identified from the Custom Transactions data). Second, they were above the US$600,000 sales threshold (to be identified from the Annual Survey of Industrial Firms data). In addition, these firms are unlikely to be intermediate (trading) firms after careful inspection of the firm names and input-output ratios.

[27]The major products for exporting are colorful dinnerware and ornamental articles of ceramics and glass such as statuettes. In the description of their main products, around 94.6 percent of them contains at least one of words such as glass and ceramics.

export volume and total number of export transactions. In addition, exports in this industry are ordinary trade, in which firms make their own decisions on production, pricing, and exporting. They are not constricted by existing partnerships with foreign companies. This situation is in contrast to many other Chinese industries that conduct processing trade.[28]

### 4.2 Reduced-Form Evidence and Patterns

The data reveals dynamic export participation patterns that are consistent with the model. First, the attrition rate of entrants is high. Define firms that began to export to the German market in a given year as a cohort. In Table 1, each column reports the percentage of firms in a cohort that exported in or after the year of entry. It shows that approximately 35 to 40 percent of the firms dropped out of the market after the first year of exporting, despite the significant sunk cost of starting exporting. In contrast, after the first year, the percentage decreases much less significantly. This is consistent with the pattern shown in Eaton et al. (2013). Aggregate demand or trade cost shocks cannot explain the large attrition rate because this pattern appears in all cohorts that entered in different years. Moreover, among firms that dropped out after the first year of exporting, only 45 percent had decreases in labor productivity at the time of exiting, while the others had increases in labor productivity. This suggests that supply-side factors may be important, but they are unlikely to be the full story of the high attrition rate.

Table 1: Percentage of exporters in different entry cohorts

| | Year of entry | | | |
|---|---|---|---|---|
| Year | 2001 | 2002 | 2003 | 2004 |
| 2001 | 100 | - | - | - |
| 2002 | 65 | 100 | - | - |
| 2003 | 64 | 66 | 100 | - |
| 2004 | 64 | 61 | 60 | 100 |
| 2005 | 50 | 64 | 60 | 64 |
| 2006 | 47 | 61 | 52 | 61 |

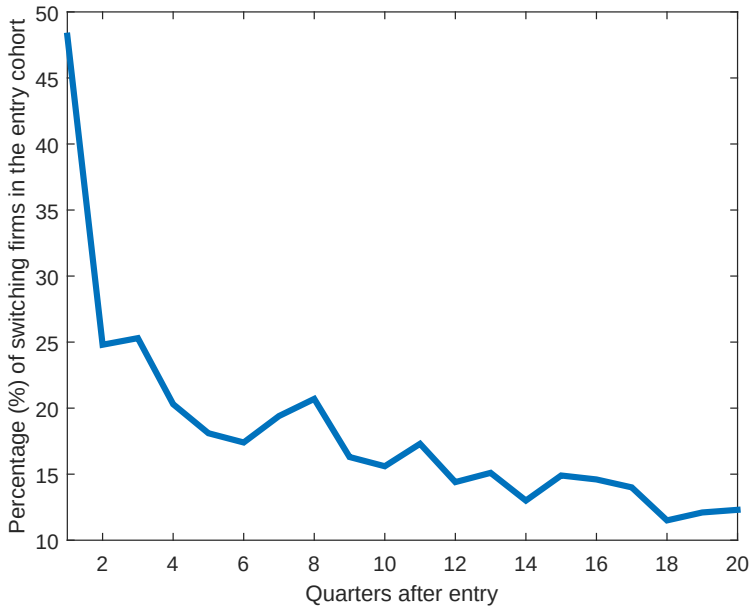
Note: Percentages are based on the total number of firms in each cohort.

Second, entrants' export participation gradually stabilizes over time. Consider a cohort of

[28] In industries conducting processing trade, firms' intermediate inputs are directly supplied by foreign partners and their final products are supposed to be shipped to foreign markets as instructed by their foreign partners. As such, these firms do not make export decisions independently and they are less likely to be affected by the uncertainty in foreign markets.

entrants that entered the German market in the last quarter of 2001.[29] Figure 1 plots the percentage of firms that switched export status (started or stopped exporting) in this cohort in each quarter. It shows that the percentage decreases and *gradually* stabilizes after about three years.[30] This is consistent with the implication of the model where firms learn their demand according to Bayes' rule. In contrast, a model with one-shot learning cannot capture this pattern, because one-shot learning implies that the switching rate drops significantly and then remains constant immediately after entry.

Figure 1: Percentage of firms that switch export status after entry




Third, reduced-form evidence shows that productivity and demand learning matter in firms' decision to export. I consider a subsample of experienced firms (i.e., firms that exported to Germany in their first year in the data set), to avoid complications of dynamic participation and initial conditions. I estimate the following linear probability model using the subsample:

$$\Pr(e_{it} = 1) = \alpha_e e_{it-1} + \alpha_{\tilde{\omega}} \tilde{\omega}_{it} + \alpha_{\bar{\zeta}} \bar{\zeta}_{it-1} + \alpha_n n_{it-1} + \alpha_N N_{it-1} + \alpha_{N2} N_{it-1}^2 + D_t + \epsilon_{it}.$$

In this regression, $\tilde{\omega}_{it}$ is the productivity measure. $\bar{\zeta}_{it-1}$ is the average demand signal received by

[29] That is, these firms did not export to Germany for about two years (2000 and 2001). The reason of choosing this cohort is to reduce the potential error of calling an experienced firm an entrant.

[30] This pattern is not limited to the German market or the firm cohort that began to export in a given year. It also appears in other export cohorts and export markets.

firm $i$ over the sample period up to year $t-1$, as defined in equation (16). $n_{it-1}$ is the number of orders received in year $t-1$. $N_{it-1}$ is the *accumulated* number of orders received over the sample period up to year $t-1$, which is a proxy for export experience. $D_t$ is a set of year dummies. The regression results, as reported in Table 2, are robust to different specifications and probability models. As expected, the positive estimates of $\alpha_{\bar{\zeta}}$ and $\alpha_{\tilde{\omega}}$ show that firms with higher demand and productivity are more likely to export. These results are consistent with recent findings on the importance of demand and cost drivers of exporting (e.g., Eckel et al., 2015; Hottman et al., 2016; Aw and Lee, 2017; Roberts et al., forthcoming). Further, the positive estimate of $\alpha_n$ suggests that firms that receive more orders per year are more likely to export. But more importantly, the estimate of $\alpha_N$ is negative. This implies that firms that have accumulated more experience face less demand uncertainty, and they are more likely to exit the foreign market. This finding corroborates the role of export age in product switching in Timoshenko (2015) and the role of firm age in firm growth in Arkolakis et al. (2017). It shows that uncertainty (in addition to productivity and demand) plays an important role in export participation. In particular, based on the results in the last column in Table 2, one-standard-deviation increases in productivity, demand, and export experience change the export probability by 3.6, 6.6, and -30.3 percentage points, respectively.

Finally, the relative importance of productivity and demand uncertainty differs for experienced firms and potential entrants to the foreign market. I define a firm as an experienced firm if it exported to Germany in its first year in the data set; otherwise, it is defined as a potential entrant. A potential entrant might choose to export in any subsequent year, although it did not export in the first year. I compute the percentage of exporting firms conditional on different combinations of productivity (measured by $\tilde{\omega}$) and demand (measured by $\bar{\zeta}$) levels.[31] Table 3 reports the results separately for experienced firms (in the upper panel) and potential entrants (in the lower panel). Overall, firms with higher productivity or higher demand are more likely to export in both groups. These findings are consistent with the regression results in Table 2. Interestingly, the comparison between the two groups of firms shows that demand plays a much more important role for potential entrants than for experienced firms. For example, conditional on high productivity, high demand

[31] For example, the top-left number, 41.9, means that in the firm group with low $\tilde{\omega}$ and low $\bar{\zeta}$, 41.9 percent of them export. "High" and "low" mean higher than the top one-third in the quantile and lower than the bottom one-third in the quantile, respectively.

Table 2: Results of the export participation regressions

| | (1) Linear | (2) Linear | (3) Linear | (4) Probit | (5) Logit |
|---|---|---|---|---|---|
| $\alpha_e$ | 0.495*** | 0.510*** | 0.423*** | 0.758*** | 1.040** |
| | (0.073) | (0.073) | (0.075) | (0.266) | (0.471) |
| $\alpha_{\tilde{\omega}}$ | 0.130* | 0.132* | 0.106 | 0.604** | 1.054** |
| | (0.072) | (0.071) | (0.069) | (0.294) | (0.505) |
| $\alpha_{\bar{\zeta}}$ | 0.052*** | 0.051*** | 0.042*** | 0.157*** | 0.275*** |
| | (0.012) | (0.012) | (0.012) | (0.043) | (0.072) |
| $\alpha_n$ | | | 0.015*** | 0.169*** | 0.363*** |
| | | | (0.003) | (0.050) | (0.103) |
| $\alpha_N$ | | | -0.001** | -0.047* | -0.091** |
| | | | (0.001) | (0.027) | (0.045) |
| $\alpha_{N2}$ | | | | 0.001* | 0.002* |
| | | | | (0.001) | (0.001) |
| Year | No | Yes | Yes | Yes | Yes |
| Observations | 443 | 443 | 443 | 443 | 443 |
| $R^2$ | 0.285 | 0.291 | 0.330 | 0.335 | 0.343 |

Note: Standard errors in parentheses.
* $p < .10$, ** $p < .05$, *** $p < .01$.

Table 3: Percentage of exporters, by group

| Experienced firms | Low $\tilde{\omega}$ | High $\tilde{\omega}$ |
|---|---|---|
| Low $\bar{\zeta}$ | 41.9 | 64.5 |
| High $\bar{\zeta}$ | 65.5 | 90.9 |
| **Potential entrants** | **Low $\tilde{\omega}$** | **High $\tilde{\omega}$** |
| Low $\bar{\zeta}$ | 8.4 | 7.4 |
| High $\bar{\zeta}$ | 12.4 | 14.7 |

increases the percentage of exporters by around 41 percent in the group of experienced firms.[32] In contrast, for potential entrants, the increase is more than doubled. Presumably, this difference is due to the greater uncertainty faced by potential entrants. It motivates me to allow firms in the two groups to have different initial beliefs in estimating the structural model in Section 5.2.

In sum, the patterns show that both productivity and Bayesian demand learning contribute to the observed export dynamics. In order to investigate their individual roles, I turn to the estimates of the structural model parameters and counterfactual exercises in the following sections.

[32] That is, (90.9-64.5)/64.5 = 0.41.

# 5 Empirical Results

## 5.1 Estimates of Static Parameters

The static parameters include the parameters in the marginal cost function, productivity evolution, and the demand elasticities in the domestic and foreign markets. Table 4 shows the estimates of the key marginal cost and productivity parameters. Coefficient $\gamma_k$ measures the elasticity of capital stock in the marginal cost. The negative estimate suggests that larger firms have greater cost advantage. More importantly, $\hat{g}_1 = 0.872$ in the AR(1) productivity evolution implies that productivity evolves persistently. A large part (but not all) of productivity is carried over from one year to another. At the same time, the standard deviation of the innovation term ($\sigma_\epsilon$) of the productivity process is estimated as 0.148. It implies that there is a significant, unexpected productivity shock that shifts firms' productivity over time. Thus, it is crucial to take the evolution of productivity into account in studying the role of the demand learning process.

Table 4: Estimates of the marginal cost and productivity evolution parameters

| Parameter | $\gamma_k$ | $g_0$ | $g_1$ |
|---|---|---|---|
| Estimate | $-0.053$ | 0.041 | 0.872 |
| S.E. | (0.006) | (0.008) | (0.005) |

Although the dynamic estimation focuses on exports to Germany, the estimate of domestic demand elasticity is needed to recover the productivity measure, as shown in Section 3.1. The demand elasticity of the German market is needed to construct the demand signals. Thus, I group other markets into a single market with elasticity $\eta^{oth}$ and equation (33) becomes[33]

$$TVC_{it} = (1 + \frac{1}{\eta^D})R^D_{it} + (1 + \frac{1}{\eta^{DEU}})X^{DEU}_{it} + (1 + \frac{1}{\eta^{oth}})X^{oth}_{it} + \upsilon^c_{it}. \tag{38}$$

The results in Table 5 show the demand elasticity estimates, which are in line with those in the Chinese footwear industry in Roberts et al. (forthcoming). The demand elasticity of the German

[33]Following Aw et al. (2011), I assume that the error $\upsilon^c_{it}$ is uncorrelated to the revenues, so I can estimate the equation using an ordinary least squares regression. Without this assumption, the estimate is biased. However, I experimented with different values of $\eta^{DEU}$, ranging from -3 to -2 (similar to the estimated elasticities in the Chinese footwear industry in Roberts et al., forthcoming, in which a set of instruments is used). I found that the dynamic estimation result is robust to the change of $\eta^{DEU}$.

market is estimated to be lower than that of "other markets". This finding is also consistent with the finding in Roberts et al. (forthcoming) that higher-income destinations have relatively lower demand elasticities.[34]

Table 5: Estiamtes of demand elasticities

| Parameter | $\eta^D$ | $\eta^{DEU}$ | $\eta^{oth}$ |
|---|---|---|---|
| Estimate | -4.448 | -2.212 | -4.517 |
| S.E. | (0.052) | (0.465) | (0.123) |

Further, by estimating equation (34), I find that the standard deviation of signal noise $u_{it}$ in the German market is $\hat{\sigma}_u = 1.690$. This value is very close to the calibrated value of the standard deviation of signal noise in Arkolakis et al. (2017), where a dynamic model of learning and firm growth is calibrated using Colombian plant-level data.

## 5.2 Estimates of Dynamic Parameters

It would be ideal to have the entire history of exports of all firms to estimate the dynamic model. But the data set only includes exports starting from 2000. Because firms' exports prior to 2000 are not observed, there is a need to deal with the problem of initial conditions. In general, a firm's export participation status in the pre-sample period influences its decisions in the sample period. Without controlling for the initial conditions, the estimation may produce biased estimates. As pointed out by Heckman (1981), the initial conditions problem can be ignored under one of two assumptions: the initial conditions are truly exogenous, or the dynamic process is stationary starting from the initial observation of the data. In this application, neither of the assumptions is reasonable. Firms' initial export decisions observed in the data depend on their export participation in the previous period, and the demand learning process is not initially stationary. To address this problem, I use the firms' state variables in their initial year in the sample period to control for the initial conditions. Importantly, I allow firms to hold different initial beliefs.[35]

[34] The United States granted Permanent Normal Trade Status to China in January 2002. The new trade status might have affected demand elasticity $\eta^{oth}$, because I include the United States into "other markets". However, I tested a specification of equation (38), where $\eta^{oth}$ can be different before and after 2002. I found the estimates quantitatively similar to the estimates of equation (38).

[35] The initial belief is the belief held by a firm in its initial period. A firm's initial period is defined as the period when the firm first appeared in the Annual Survey of Industrial Firms data.

In principle, it is possible to write the mean and standard deviation of the initial belief as functions of observable characteristics (such as firm age and size) and estimate the associated parameters. However, doing so would dramatically increase the number of dynamic parameters and the computational burden of the estimation.[36] In contrast, a firm's export status in its initial year indicates whether the firm has gained some knowledge of its foreign demand. Thus, its export status in the initial year serves as a better predictor about the firm's initial knowledge of demand in the pre-sample period. As such, I divide the firms into two groups: potential entrants to the German market and experienced firms, according to whether they exported to Germany in their initial year, as described in Section 4.2.[37] By allowing the two groups of firms to have different initial beliefs, this approach involves estimating fewer parameters in the dynamic model. Specifically, I assume that potential entrants and experienced firms have initial beliefs $N(m_0^p, \sigma_0^p)$ and $N(m_0^e, \sigma_0^e)$, respectively. These parameters are flexibly estimated as a part of the dynamic parameters. The initial beliefs not only affect export participation in the initial year, but also influence all subsequent years via the Baysian learning process. The distribution of export costs is assumed to be the same for the two groups.

Table 6: Estimates of the belief and trade cost parameters

| Parameter | $m_0^p$ | $m_0^e$ | $\sigma_0^p$ | $\sigma_0^e$ | $c^f$ | $c^s$ | $\delta$ |
|---|---|---|---|---|---|---|---|
| Estimate | $-3.654$ | $-1.828$ | 0.459 | 0.297 | 0.204 | 2.707 | 0.880 |
| S.E. | (0.059) | (0.027) | (0.009) | (0.006) | (0.073) | (0.086) | (0.038) |

The estimates of initial beliefs in Table 6 suggest that experienced firms tend to have a higher initial expectation and face less uncertainty compared with potential entrants. This is intuitive. Unlike potential entrants, experienced firms may have operated in the foreign market for a long time and have learned a lot before the sample period. The estimate of the standard deviation of signal noise, $\hat{\sigma}_u = 1.690$, is significantly larger than the standard deviations of initial beliefs. This

[36] In addition, in a probit regression, I found that the initial conditions (decisions of export participation) were only insignificantly related to firm age, capital stock, and ownership. This suggests that these variables are not good predictors of firms' initial demand knowledge or initial beliefs.

[37] In effect, this is to use the export status in the initial year of the observed sample to approximate the experience of a firm in the pre-sample period. Such an approximation is appropriate because of the high persistence of export participation, which is well-documented in the literature. Indeed, in the sample, only 3.6 percent of exporters were new entrants in 2001. This suggests that the potential error of defining a firm as experienced according to its export status in the initial year is far from severe. In Online Appendix B, I provide further discussion and show that the dynamic estimation result is robust to stricter definitions of experienced firms.

implies that the learning is slow. In particular, it takes around 18 transactions (signals) for a fresh entrant to reach the same level of uncertainty as an experienced firm.

The estimates of the export cost parameters are reported in Table 6 as well. These estimates suggest that the unconditional means of the fixed and sunk costs are US$24,700 and US$327,300, respectively. These findings imply that the cost of exporting prevents all but only firms with high productivity or high demand expectation from exporting.

The estimates of the order process parameters are reported in Table 7. The estimate of $\psi_n$ is 0.896, which implies that firms with more orders in the last year are likely to secure more orders this year. It is important to take this into account to control for the different speeds of learning across firms in quantifying the role of the learning process.

Table 7: Estimates of the order process parameters

| Parameter | $\phi_0$ | $\phi_n$ | $\rho$ |
|---|---|---|---|
| Estimate | 0.217 | 0.896 | 3.523 |
| S.E. | (0.115) | (0.046) | (0.420) |

### 5.3 Productivity, Demand Learning, and Export Experience

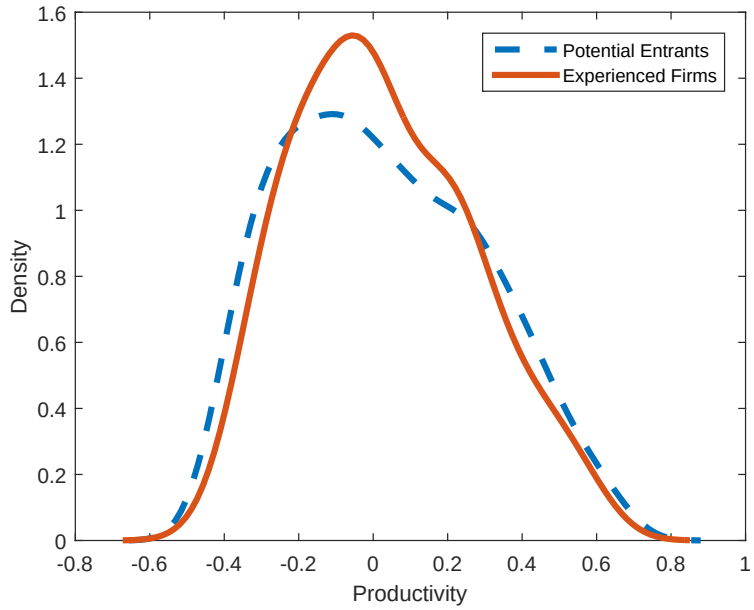
The literature documents substantial firm heterogeneity in productivity and foreign demand, which influence firms' decision to export. In an environment of uncertain foreign demand, I show that both demand belief and productivity are positively associated with export experience. More importantly, the association between demand belief and export experience is stronger.

Figure 2 shows the kernel densities of productivity for potential entrants and experienced firms.[38] Although there is significant productivity heterogeneity within each group, the densities almost overlap with each other. The average productivity of experienced firms is higher than that of potential entrants by around 2.30 percent. Statistically, this is not significant at the 1 percent significance level. Economically, this finding implies that experienced firms are only slightly more profitable in the domestic market than potential entrants on average.

In contrast, Figure 3 shows the kernel densities of the demand expectation, which is substantial

[38] Productivity estimates across firms and over time are pooled together to obtain the densities. The level of productivity is normalized to a common reference point for comparison.

Figure 2: Density of productivity, by group

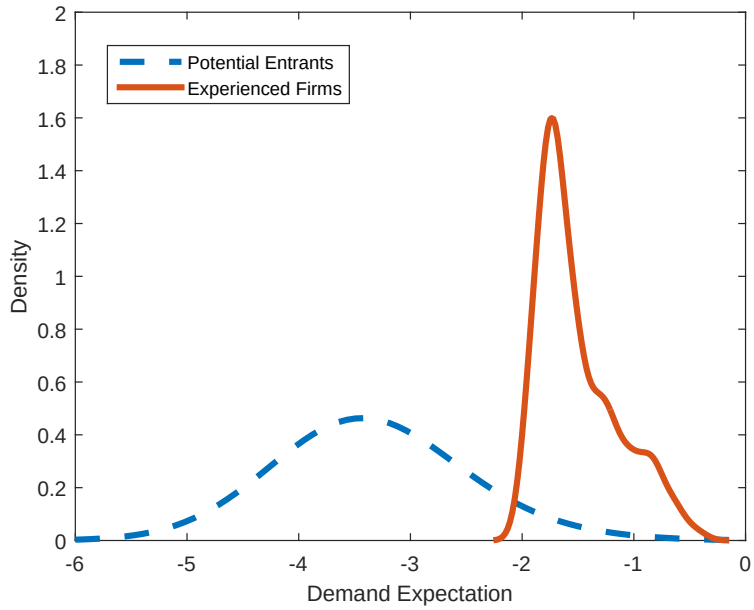



heterogeneous within each group.[39] Consistent with their initial beliefs, the demand expectations of experienced firms are higher and more concentrated compared with those of potential entrants. More importantly, the average demand expectation of experienced firms is significantly higher (at the 1 percent significance level) than that of potential entrants. Of course, the way of grouping (based on export status in the initial year) implies that firms with higher productivity or higher demand expectation are classified as experienced firms, and the figures confirms this. Beyond that, the comparison shows that although experienced firms are superior to potential entrants in both foreign demand and productivity, foreign demand is the dominant difference between the two groups.

Table 8 provides a more detailed comparison according to firm export experience. I use the accumulated number of export transactions to approximate export experience. Accordingly, I group firms by different export experience. Table 8 shows that the means of productivity $\tilde{\omega}$, expected demand $m$, and uncertainty $\sigma$ of each group. The firms with more experience have higher productivity and higher expected demand, and they face less uncertainty. Nonetheless, the average productivities across experience groups are not statistically different at the 1 percent significance level. In contrast, the average expected demands and uncertainties differ across groups at the 1 percent significance level. In addition, the correlation coefficient between productivity

[39]The demand expectation is measured as $m + \sigma^2/2$, because $E(\exp(\xi)) = \exp(m + \sigma^2/2)$.

Figure 3: Density of demand expectation, by group



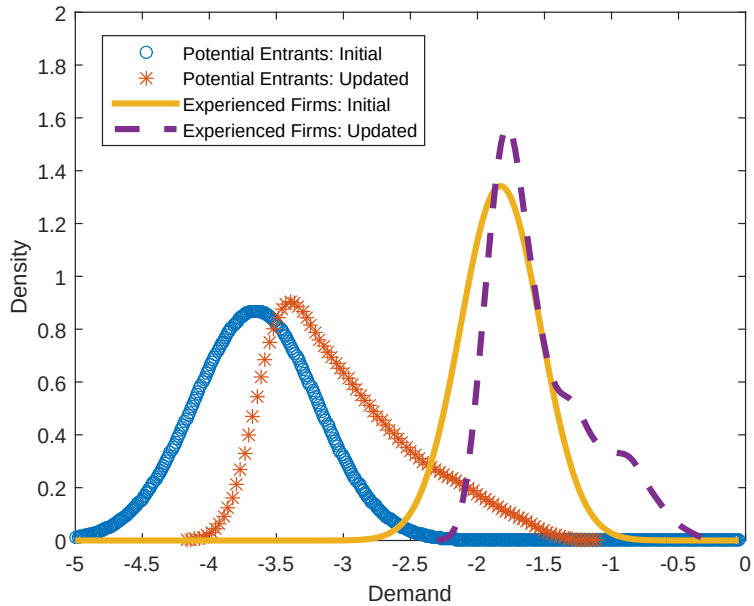

and experience is only 0.10, while expected demand and uncertainty are strongly correlated to experience, with correlation coefficients 0.58 and -0.65, respectively. Overall, the results show that although productivity is positively associated with export experience, high foreign demand with low uncertainty serves as the main predictor of success in the German market.

Table 8: Average productivity, expected demand, and uncertainty, by export experience

| Accumulated Tx ($N$) | $\tilde{\omega}$ | $m$ | $\sigma$ |
|---|---|---|---|
| 0 | 0.120 | -3.654 | 0.459 |
| 1∼10 | 0.145 | -2.451 | 0.354 |
| 11∼20 | 0.190 | -1.527 | 0.271 |
| 21+ | 0.235 | -1.264 | 0.223 |

Finally, it is worth noting the difference between initial demand beliefs and posterior demand densities. For each group of firms, the posterior demand density is constructed from the expected demands (i.e., $m_{it}$, pooled from all years). Figure 4 shows that the posterior demand densities are less dispersed, and, importantly, they are to the right of the initial beliefs. This finding suggests that some firms, which initially underestimated their demand, updated their expectations to higher levels after exporting. This is true for both groups of firms, but it is more significant for potential entrants. In particular, for potential entrants, the average expected demand is higher than that of

Figure 4: Initial beliefs and posterior demand densities, by group




the initial belief by 27.8 percent. Thus, if potential entrants export because of favorable policies (e.g., trade cost reduction), then some entrants may eventually survive in the foreign market if they learn that their demand is high. This directly motivates the counterfactual exercise in the next section, which investigates the long-run indirect effect of trade cost reduction on promoting exporting in an environment of uncertain foreign demand.

# 6 Counterfactual Analysis

This section employs the estimated model to explore the roles of the evolution of productivity and demand learning in influencing export participation. The analysis consists of two sets of counterfactual exercises. In the first set of exercises, I investigate the relative importance of the two processes for different groups of firms. In the second set of exercises, I explore how demand learning amplifies the effect of trade cost reduction in an environment of uncertain foreign demand.

## 6.1 Relative Importance: Demand Learning vs. Productivity Evolution

In this set of exercises, I consider two restricted scenarios. In the first scenario, I shut down the demand learning process in the model. That is, each firm knows its foreign demand and does not

face uncertainty. In contrast to the full model, the firms in the restricted model do not update their beliefs after exporting. The other components are the same as those in the full model. I solve the Bellman equation in this restricted model and average the predicted export participation decisions over 200 simulations. The predicted percentages of exporters among potential entrants and experienced firms are shown in Table 9. The table compares the results with those from the full model (the first row in Table 9). Overall, the percentage of exporters decreases by 33.0 percent, suggesting that uncertainty due to unobserved foreign demand is important in firms' decisions of export participation. This finding is complementary to the findings of Dickstein and Morales (2015), who measure the observable information set of firms when they make export decisions. More importantly, the percentage of exporters among potential entrants decreases by 50.0 percent, while for experienced firms it only decreases by 7.7 percent. The sharp difference implies that demand learning has a stronger effect for potential entrants. This finding is consistent with the result that potential entrants face greater uncertainty compared with experienced firms. It also echoes the reduced-form evidence on the relative importance of demand between the two groups of firms in Section 4.2.

In the second scenario, I shut down the evolution of productivity in the model. That is, I fix the productivity of each firm at the level of its initial year. This eliminates the change in productivity in the time dimension within firms. This scenario still keeps productivity heterogeneity across firms. The other components are the same as those in the full model. After resolving and simulating the restricted model, I report the predicted percentages of exporters in Table 9. In contrast to the full model, overall the percentage decreases by 9.3 percent. This is because that productivity in this industry grows over time. Fixing firms' productivity at their initial levels implies lower productivity in the future on average compared with the full model. Consequently, this reduces the number of exporters. Importantly, the percentage of exporters among potential entrants decreases by 8.1 percent, which is significantly less than the decrease in the first scenario (50.0 percent). In contrast, for experienced firms, the percentage decreases by 12.1 percent, which is greater than that in the first scenario (7.7 percent). This comparison implies that productivity evolution is the major driving force of export participation for experienced firms, but for potential entrants, demand learning plays a more important role.

The analysis complements a broad set of papers that focus on productivity or demand learning

Table 9: Relative importance: Demand learning vs. productivity evolution

| | Percentage of Exporting Firms among | | |
|---|---|---|---|
| | Potential Entrants | Experienced Firms | All Firms |
| Full Model | 14.8 | 72.6 | 21.5 |
| No Demand Learning | 7.4 | 67.0 | 14.4 |
| No Productivity Evolution | 13.6 | 63.8 | 19.5 |

as a determinant of exporting at the firm level. Traditionally, the literature (i.e., Melitz, 2003; Bernard et al., 2003; Helpman et al., 2004; Eaton et al., 2011; Arkolakis, 2011; Aw et al., 2011) has studied how firm-level exports are driven by heterogeneous firm productivity. A growing strand of trade models has begun to emphasize the roles of demand uncertainty and learning in export markets (Rauch and Watson, 2003; Freund and Pierola, 2010; Albornoz et al., 2012; Nguyen, 2012; Eaton et al., 2013; Timoshenko, 2015; Berman et al., forthcoming). A missing block is to take the evolution of productivity at the firm level empirically into account in studying the role of the learning process. The above analysis fills this gap. It corroborates the literature on productivity and uncertain demand as determinants of export participation. The analysis further shows the different relative importance of productivity and demand uncertainty across firms. In this aspect, the results are also consistent with the finding of Roberts et al. (forthcoming) that demand plays a more important role than supply-side factors in driving firms to export.

### 6.2 Role of Demand Learning in Trade Cost Reduction: An Indirect Effect

The second set of counterfactual exercises is to understand the role of the demand learning process in enhancing export participation over the long run after the cost of trade is reduced. In the literature (e.g., Lincoln and McCallum, 2016), it is expected that a reduction in the cost of trade can boost export participation, especially for potential entrants. In an environment of uncertain foreign demand, this can be driven by two effects. The first one is a direct effect of self-selection. Firms with lower productivity or lower demand can enter the foreign market after the cost of trade is reduced. The second and more important effect is that firms endogenously react to what they learn from their own export experience after entering the foreign market. When entrants receive high demand signals, they update demand beliefs and may continue exporting even if they are hit by negative productivity shocks. This effect is supported by the finding in Section 5.2 that the

distributions of expected demand are to the right of the initial beliefs. However, without considering demand uncertainty, traditional analysis usually ignores the implication of the indirect effect in the long run.

Table 10: Direct and full effects of trade cost reduction: The role of demand learning

| | Exporter Percentage Point Change | | | |
|---|---|---|---|---|
| Year | 2 | 5 | 10 | 15 |
| Direct Effect (firms do not react to demand signals) | 0.13 | 0.25 | 0.31 | 0.35 |
| Full Effect (firms react to demand signals) | 1.79 | 1.16 | 0.75 | 0.73 |

Note: Numbers reflect the difference between the results of the counterfactual scenarios and those of the baseline full model (i.e., no trade cost reduction).

To investigate the indirect effect, I consider the cohort of potential entrants in 2002 (the first year after China's accession to the World Trade Organization) under three scenarios. The first scenario is the full model, in which there is no reduction in fixed cost or sunk cost. This scenario serves as the baseline case. I simulate the model over a 15-year path 200 times and record the percentage of exporting firms as well as the dynamic state of each firm.

In the second scenario, the fixed and sunk costs are reduced by 20 percent permanently. I solve the dynamic model and evaluate the probability of exporting on the *same* dynamic state as the baseline case. That is, new entrants, which are induced by the reduced cost of trade, do not respond to the demand signals they observe from their export experience. The difference between the percentage of exporters in this scenario and that in the baseline captures the direct effect after the trade cost reduction. The results are shown in the first row in Table 10. As expected, the trade cost reduction increases the percentage of exporting firms by 0.13 percentage point after two years. The increase rises to 0.35 percentage point (approximately a 10.6 percent increase relative to the baseline) by the 15th year.

In the third scenario, the fixed and sunk costs are reduced by 20 percent permanently, and I solve the dynamic model and evaluate the probability of exporting on their own simulated dynamic states (rather than on the states from the baseline as in the second scenario). Thus, in contrast to the second scenario, new entrants update their demand beliefs according to the observed demand signals. The difference between the percentage of exporters in this scenario and that of the baseline reflects the full effect (i.e., the sum of the direct selection effect and the indirect effect due to demand

learning) of the trade cost reduction, as shown in the second row in Table 10.

Overall, the reduced cost of trade indeed boosts the percentage of exporting firms due to the endogenous learning process in addition to the direct selection effect. However, the indirect effect slowly decreases over time. Comparing the second row with the first row in Table 10, the difference (measuring the indirect effect) is reduced from 1.66 percentage points in the second year to 0.38 percentage point in the 15th year. Notably, this difference still accounts for more than half of the full effect.[40] The decrease is intuitively due to the reduced demand uncertainty as firms become more experienced. The demand learning process plays an important role initially. But over time, its effect is almost caught up by the direct selection effect. This finding is consistent with the finding in Section 6.1 on the relative importance of demand learning between experienced firms and potential entrants. Recent literature (e.g., Eckel et al., 2015; Hottman et al., 2016; Aw and Lee, 2017; Roberts et al., forthcoming) emphasizes demand as a determinant of firms' export decisions beyond productivity. The counterfactual analysis contributes to the literature by investigating the role of the demand belief (rather than demand itself) over the long run after the cost of trade is reduced.

# 7 Conclusion

The traditional international trade literature focuses on modeling production. Recent papers have begun to emphasize the role of demand, especially the uncertainty of foreign demand. In this paper, I incorporate both into a structural model of export dynamics to empirically quantify how the evolution of productivity and the Bayesian demand learning process affect export participation separately. I use data on firm-level production and transaction-level exports to Germany in the Chinese ceramics and glass industry to estimate the structural model.

The empirical results show substantial firm heterogeneity in both productivity and demand belief. However, their relative importance differs across firms. For experienced firms, productivity is the major driving force of export participation. In contrast, for potential entrants in the foreign market, demand learning plays a more important role. I also find evidence that entrants that received high demand signals continued exporting to the foreign market. Based on these findings, a

[40] These numbers are economically significant. They account for 23.5 and 11.5 percent of the total number of exporters in the full effect scenario, respectively.

set of counterfactual exercises is conducted to demonstrate the role of demand learning in enhancing export participation after trade cost reduction. The results show that the reduction in the cost of trade has a significant impact on stimulating the export participation of potential entrants. Importantly, more than half of the increase in participation is attributed to firms' endogenous reaction to demand learning in the foreign market over the long run.

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