A Structural Model of Productivity, Uncertain Demand, and Export Dynamics*

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Abstract

This paper develops a structural model of export dynamics to empirically study how firms' market-level export decisions depend on productivity evolution and Bayesian learning about demand in foreign markets. Firms have uncertainty about foreign demand and they gradually learn about it based on the prices and quantities they observe in their individual export transactions in the Bayesian style. Firms' export decisions are dynamic and depend on the evolution of both productivity and beliefs about foreign demand. I empirically identify the role of each process in determining firm-market-level export participation and estimate the dynamic model. The identification and estimation use data on both firm shipment-level exports and firm-level production information for the Chinese ceramics industry. The empirical results indicate substantial firm heterogeneity in both productivity and demand uncertainty. Demand uncertainty is the dominant difference between potential entrants in export markets and experienced exporters. In particular, experienced exporters have higher expectations and face less uncertainty about foreign demand. Both the learning process and productivity evolution are driving forces of export participation for experienced exporters but for potential entrants the former plays a more important role. A further counterfactual exercise shows that reducing the level of uncertainty of potential entrants to that of experienced exporters causes the number of exporters to fall by 11%.

Keywords: *demand uncertainty, Bayesian learning, productivity, export dynamics*

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1 Introduction

A general finding in the recent literature using firm-level trade data is that firms' export decisions are driven by unobservable firm heterogeneity in both cost and demand dimensions (e.g., Melitz (2003); Bernard, Eaton, Jensen, and Kortum (2003); Eaton, Kortum, and Kramarz (2011); Roberts, Xu, Fan, and Zhang (2012)). More productive firms enjoy lower marginal costs in production; firms with high foreign demand earn larger market shares at given prices. Both mechanisms contribute to the heterogenous export decisions observed across firms. However, some dynamic features of exporting, such as the high attrition after the first year of exporting and gradually stabilized export decisions,[1] are not reconciled by these mechanisms but are consistent with firms' behavior when they face uncertainty about foreign demand. Indeed, firms are likely to have uncertainty about demand when they enter into unfamiliar foreign markets. For example, they may be uncertain about how foreign customers perceive the quality of their products. But if they export, they are able to learn about it from the quantities sold in the export market, after controlling for prices. As a result, demand uncertainty induces firms to export as an experiment; but whether or not to continue exporting depends on their expected demand which changes over time based on the export outcomes they observe. Essentially, firms' export decisions do not only depend on their productivity but also rely on their demand expectations which are likely to evolve endogenously as firms export.

This paper studies the role of learning mechanisms in explaining dynamic features of exporting and contributes to the vibrant area of related research (Rauch and Watson (2003); Freund and Pierola (2010); Albornoz, Calvo Pardo, Corcos, and Ornelas (2012); Nguyen (2012); Eaton, Eslava, Jinkins, Krizan, and Tybout (2013)).[2] First, I develop a single-agent infinite-horizon dynamic model of export decisions in which the firm faces uncertainty about foreign demand and gradually learns about it from its export transactions according to Bayes' rule. In particular, the size of each individual transaction, after controlling for the price, influences the firm's expectation about foreign demand; the number of transactions in each period affects the speed of learning. Second, I take the important driving force, productivity evolution, into account when I assess the role of the

[1] These patterns are shown in Section 2.

[2] Some other empirical papers study the role of uncertainty and learning in a structural model. For example, see Ackerberg (2003) for a dynamic learning model to study both "informative" and "prestige" effects of advertising, and Crawford and Shum (2005) for a study of effects of uncertainty and learning in the demand for pharmaceutical drugs.

learning process. That is, the firm's export decision depends on both productivity and its demand belief. Both of them evolve over time, but neither of them is observable to researchers. I empirically identify the role of each process in determining firm-market-level export participation and estimate the dynamic model using shipment-level exports and firm-level production data for the Chinese ceramics industry.

The recent empirical literature has documented firm-level demand heterogeneity as a determinant of firm performance (e.g., Foster, Haltiwanger, and Syverson (2008)). In the export context, Roberts, Xu, Fan, and Zhang (2012) find substantial firm heterogeneity in both the demand and cost dimensions with demand being more dispersed. My paper builds on this insight: firms face heterogenous demand in foreign markets. But the novelty is that they have uncertainty about foreign demand, thus it is the belief about demand that really influences a firm's export decision. Moreover, a forward-looking firm will expect to update its demand belief after exporting, so its demand belief endogenously evolves as the firm observes more of its own export outcomes. Specifically, I explicitly model the firm-market-specific demand faced by a firm as a demand factor. It reflects a combination of customer taste and the relative product quality difference between the firm and other suppliers, both of which introduce potential uncertainty about demand. In particular, the demand factor is used to model the intercept of the demand curve faced by the firm in the foreign market. The firm's export decision is influenced by its demand factor since a higher demand factor implies more profit at a given price. However, the firm only knows the slope but not the intercept (demand factor) of the demand curve,[3] and makes the export decision based on its belief about the demand factor. This belief can be updated based on the firm's own export experience in the foreign market. To be specific, the quantity sold in each individual transaction reflects a signal of the demand factor, after controlling for the price. The signal is noisy because it contains an idiosyncratic demand shock which is not observed by the firm. Due to the noise, demand uncertainty is not completely resolved immediately after just one transaction. Nonetheless, the firm is able to update its belief at the end of each period based on the received signals in the Bayesian style. This posterior belief then guides

[3] Aw, Roberts, and Xu (2011) find that the slope of the demand curve in the foreign market, i.e., demand elasticity, is virtually identical to that of the domestic market for Taiwanese electronics industry. This result is also found for Chinese industries in this paper. Thus, compared with the intercept, firms face far less uncertainty about the slope of the demand curve in the foreign market since they have been operating in the domestic market for a long time.

the firm to make its export decision in the next period. In this way, the export decision and the belief updating are endogenously correlated: the firm makes export decision based on its current demand belief knowing that its exports in this period will update the demand belief which will further affect its future export decisions.

However, the assessment of the role of the learning process will potentially be biased if another driving force of export dynamics, productivity evolution, is ignored. Productivity has been recognized as a distinct cost-side heterogeneity in studies of firm performance and survival (e.g., Baily, Hulten, Campbell, Bresnahan, and Caves (1992); Baldwin and Gorecki (1998)),[4] as well as in explaining firm-level export participation (e.g., Melitz (2003); Bernard, Eaton, Jensen, and Kortum (2003); Helpman, Melitz, and Yeaple (2004)): more productive firms have lower marginal costs and tend to enter the export market while less productive firms only serve the domestic market. In particular, Eaton, Kortum, and Kramarz (2011) show that over half the variation across firms in market entry can be attributed to productivity heterogeneity. In the time series dimension, Aw, Roberts, and Xu (2011) find that firm choices of R&D and exporting have a positive effect on the firm's future productivity, which in turn drives more firms to self-select into both activities. Thus, without controlling for the change of productivity at firm-level, it is likely to attribute the effect from the productivity shock to the role of the learning process.[5] So in my model I take care of this issue. Firms' export decisions depend on two types of firm heterogeneity: productivity and the demand belief. But this places a challenge in identifying the role of the learning process from the effect of productivity evolution, since both of them are unobservable to researchers.

To identify the role of each process, I rely on two data sets. The first one is the Chinese Annual Survey of Manufacturing. It provides firm-level production information: employment, labor and material expenditures, capital stock, and domestic revenue. The second one is the Chinese Monthly Customs Transactions. It contains shipment-level exports: the export destination, quantity and price of each shipment, and shipment month. The identification strategy uses the insight that domestic revenue is only affected by productivity while export participation is influenced by both productivity and the demand belief. Thus, I utilize data from the domestic market to recover the

[4]See Bartelsman and Doms (2000), and Syverson (2011) for reviews.

[5]Some other papers also illustrate the importance of productivity evolution in export decisions. For example, see Irarrazabal and Opromolla (2006).

time-varying productivity for each firm. Meanwhile, shipment-level exports contain information on how Bayesian learning occurs and allow me to recover a market-specific demand belief for each firm in each period. A model with only productivity heterogeneity predicts more productive firms export. While in my model with the two-dimensional heterogeneity, firms face greater uncertainty about demand may also export because of the larger option value of learning, even if their productivity is not high. In turn, with both productivity and demand beliefs recovered, the cross-sectional and time series patterns of export decisions identify the role of each driving force.

The identification strategy suggests a structural estimation approach with two stages. In the first stage, I utilize the data from the domestic market to recover firm-level productivity, its evolution process, and firm-level marginal cost. Then I take the quantities in individual shipments after controlling for prices as demand signals received by firms and use them to update firms' demand beliefs in each period. In the second stage, I estimate an discrete choice model of export participation via the Maximum Likelihood Method with the Nested Fixed Point Algorithm as in Rust (1987). The major estimation difficulty comes from the internal calculation of the value function in each evaluation of the likelihood. Since the belief enters the value function as a two-dimensional state variable *in addition* to productivity and the aggregate demand/cost shifter, the number of state variables makes it time-consuming to use the traditional method of value function iteration to derive the underlying value function. To solve this problem, I follow the method in Nagypál (2007) to compute the value function efficiently. I first calculate the value function without uncertainty. Then I use it as an approximation of the value function with a low level of uncertainty. This in turn allows me to compute the value function with arbitrarily greater uncertainty by backward induction.

In the empirical estimation, I focus on the exports of the Chinese ceramics industry to Germany from 2000 to 2006.[6] These firms export colorful dinnerware and ornamental articles of ceramics, such as statuettes. This industry fits the study purpose well. In this industry, most export transactions are ordinary trade, in which firms make their own decisions of production, pricing, and exporting, without facing constraints from existing contracts with foreign companies (as in processing trade). As a result, demand uncertainty, which comes from how foreign customers perceive the appeal of the product, is potentially an important issue to consider when firms make export decisions. To control

[6] The extension of the estimation to other destinations and industries is currently in progress.

for the initial condition, I divide firms into two groups, potential entrants in the foreign market and experienced exporters, according to their export status in year 2000. I allow the two groups to hold different beliefs when they first show up in the data set. The empirical results indicate substantial firm heterogeneity in both productivity and demand uncertainty. Demand uncertainty is the dominant difference between potential entrants and experienced exporters. In particular, experienced exporters have higher expectations and face less uncertainty about demand compared with potential entrants. This is reasonable, since experienced exporters may have operated in the foreign market for a long time and have learned a lot before 2000. I also find that the value difference between the decisions to export and not to export falls as demand uncertainty is resolved. This mechanism contributes to the observed high attrition after the first year of exporting.

Using the estimated model I conduct two counterfactual exercises to quantify the roles of productivity evolution and the learning process in determining firm export participation. In the first analysis, I shut down either the evolution of productivity or belief updating in order to evaluate how export participation is influenced by that process. The comparison of the two scenarios shows that, for experienced exporters, both evolutions significantly influence export participation while the learning process has a larger impact for potential entrants. In the second analysis, I experiment with the prior belief of potential entrants to study how it affects export participation. I find that reducing the level of uncertainty of potential entrants to that of experienced exporters causes the number of exporters to fall by 11%. While increasing the expectation of potential entrants to that of experienced exporters doubles the number of exporters. If potential entrants have the same prior belief as experienced exporters, then the percentage of exporters increases from 26% to 40%.

This paper is organized as follows. Section 2 motivates the paper by documenting dynamic features at firm-market level for Chinese manufacturing industries. Section 3 develops a structural model of export dynamics that incorporates heterogeneity in both productivity and Bayesian learning. Section 4 outlines an identification and estimation strategy. Section 5 describes the data sources for the empirical estimation. The estimation results are shown in Section 6. Section 7 conducts counterfactual exercises. I conclude in Section 8 with discussions for future work.

2 Dynamic Features of Exporting at Firm-market Level

This paper is empirically motivated by dynamic features of firm-market-level export participation patterns that are not easily reconciled by productivity evolution but are consistent with a model of uncertainty and Bayesian learning about foreign demand. The features presented in this section are based on the Chinese Monthly Customs Transactions data set.[7] This data set includes all export shipments of Chinese firms from 2000 to 2006. Each shipment contains export value, quantity, 8-digit HS code, shipment month, destination market, and firm identification number.

First, a large percentage of firms drop out after the first year of exporting to a market. I follow Eaton, Eslava, Jinkins, Krizan, and Tybout (2013) and consider the export cohorts that began exporting to the U.S. in a particular year after at least one year of no exporting. In Table 1, each column reports the percentages of firms within each cohort that chose to export after the year of entry. The pattern shows that there is a significant percentage (around 30%) of firms that drop out of the U.S. market after the first year of exporting. However, conditional on survival in the second year, the percentage of exporters stays roughly the same. Aggregate demand shocks cannot explain the large attrition rate after the first entry, since the high attrition appears in all cohorts that entered in different years and in other destinations. Did it result from a negative firm-level productivity shock? Using firm-level production data, I find that, for firms that dropped out after the first year of exporting, 45% of them were experiencing decreases in relative productivity (measured as output per worker) while 55% of them were experiencing increases in relative productivity. Thus, although it is possible that productivity evolution plays a role, it is unlikely that it alone can capture the entire significant attrition.

Second, the export decisions of new exporters *gradually* become stable over time. To ensure that the firms I am looking at are more likely to be new exporters in the U.S. market, I consider the firms that started to export to the U.S. in the last quarter of 2001 but did not export to the U.S. prior to that. I calculate the percentage of these firms that switched their export status (from export to not to export, or vice versa) in each quarter after 2002. Figure 1 shows that overall this percentage becomes smaller over time. This implies that new exporters' export decisions become more stable

[7] The patterns are for medium and large manufacturing firms that also show up in the Chinese Annual Survey of Manufacturing.

over time. More importantly, the percentage *gradually* becomes stable after several continuing significant drops. This pattern is not limited to the U.S. market or the firm cohort that began to export in a particular year, but is a universal feature that also appears in other export cohorts and in other destinations. Productivity evolution can account for the stabilized export participation, but it alone is not capable to capture the continuing significant drops in the beginning. Nonetheless, this feature is consistent with an environment of uncertainty and Bayesian learning about demand, in which firms update their beliefs about foreign demand based on their export outcomes.[8]

These dynamic features show that it is possible that both productivity evolution and Bayesian learning about demand contribute to the observed patterns, but neither of them alone can explain all patterns. This motivates me to incorporate both processes as driving forces to reconcile these patterns and study how they influence firm-level export dynamics separately.

3 The Model

In this section, I develop a structural model of firm-market-level export dynamics with heterogenous firms learning about demand curves they face in foreign markets. A novelty of the model is that it considers how the two processes, productivity evolution and the evolution of the demand belief, affect a firm's export participation. In particular, the firm learns about its foreign demand from the prices and quantities it observes in its export transactions in the Bayesian style, and in turn the firm's belief about foreign demand endogenously evolves as it exports. I first provide an overview of the model then specify the details.

3.1 An Overview

Consider an industry with I single-product firms and an infinite-period horizon. There are a total of J foreign markets in addition to the domestic market. Firms are heterogenous in both the cost and demand dimensions. In the cost dimension, firms are different in productivity, a measure of efficiency in converting input to output. Higher productivity implies lower marginal cost in production. In

[8]Potentially, a simpler model without Bayesian learning is to allow the one-shot learning and serially correlated idiosyncratic demand, which is observed by firms but is unobservable to researchers. However, such a model will predict the switching percentage to drop very shapely after the first entry and be stable afterward, which is not consistent with the observation in Figure 1.

the demand dimension, firms are heterogenous in the product quantity demanded at a given price, which results from a combination of customer tastes and the relative product quality difference between the firm and international competitors. I refer to this heterogeneity, which is not captured by the aggregate demand shifter, as the firm's demand factor. The demand factor is allowed to be different across foreign markets for each firm, in order to capture different consumer tastes and international sellers in different markets. For each firm, productivity evolves exogenously from one period to another,[9] but the demand factor in each market is constant.

In each period t, each firm makes decisions of production, pricing, and export participation in each foreign market and the domestic market.[10] Two types of information are crucial for it to make these decisions. The first one is its productivity level, and the second is its demand factor in each foreign market. At the beginning of each period t, firms observe the realization of their productivity ω_{it}. However, in contrast to the current literature (e.g. Roberts, Xu, Fan, and Zhang (2012)), this paper assumes the foreign demand factor is firm-market specific and unknown to firms. As a result, each firm makes the export decision based on its current belief about its demand factor in each foreign market at the beginning of period t.

For the sake of simplicity, I assume that the firm's export decisions for different markets are independent.[11] If the firm decides to export to market j, then it pays either a fixed cost (c_{it}^{fj}) or a sunk entry cost (c_{it}^{sj}), depending on its export status in market j in the last period $t-1$. Once in market j, firm i receives (and fulfills) n_{it}^{j} orders from customers in that market. By observing the quantity demanded at a given price in *each* transaction, the firm can learn about its demand factor in market j. In particular, the belief about the demand factor is updated at the end of period t in the Bayesian style, based on these observations. However, if the firm decides not to export in

[9] If firms endogenously choose productivity, by investing in R&D for example, then more productive firms are more likely to conduct R&D (Doraszelski and Jaumandreu (2013)). The assumption of exogenous productivity evolution will attribute the impact from R&D to the high persistence of productivity. I abstract from the endogenous productivity evolution because of lack of R&D data in this paper. I discuss the future research along this direction in the conclusion section.

[10] I abstract from the decision on enter or exit production, and assume that each firm in production will serve the domestic market.

[11] Independence of export decisions does not necessarily imply the export participation outcomes are independent. Actually, the outcomes are correlated because of productivity of manufacturing goods for different markets is the same within the firm. Similar independence assumption is employed in the recent literature (e.g., Eaton, Kortum, and Kramarz (2011)).In the industry I will consider, the correlation of export participation in two destination is only around 0.1.

market j, then it does not pay the fixed/sunk cost, and there is no export profit generated from that market. More importantly, the belief regarding the demand factor in that market will not be updated at the end of period t because no transaction has happened (or been observed).

Several points need to be clarified. First, the learning processes of different firms in different markets are assumed to be independent. This assumption is motivated by data. This paper study the kind of industries in which firms mainly conduct ordinary trade and demand uncertainty is potentially an important issue when firms make export decisions. For these industries, including ceramics industry, fiber optics industry, and many others, it is common that firms have large sales in one destination but no sale in another. For example, the correlation coefficients between export revenue across destinations are on average 0.15 in the ceramics industry. This implies the firm export revenue is not highly correlated in different destinations. This is consistent with the assumption that the underlying demand factor is firm-market specific, thus the independent learning assumption is reasonable, especially for markets with great cultural and geographic distinctions.[12] Second, it is feasible to consider that productivity evolution is affected by export participation; however, export participation in a *single* market usually has an insignificant effect on productivity evolution.[13] Thus, I do not consider such productivity gains from exporting in this paper. However, the belief about the demand factor in each market is endogenously related to the export participation in that market. Third, the (period by period) export growth at firm level is captured by the increase in the number of orders that the firm receives in a period (i.e., n_{it}^j will increase if the firm keeps exporting). This captures the possibility that the stock of customers who are familiar with the firm or the firm's distribution network is increasing over time if the firm keeps exporting.

The timing is summarized as follows:

t — Observe ω_{it}, b_{it-1}^j, c_{it}^{fj}, c_{it}^{sj} — Choose e_{it}^j — Receive, fulfill n_{it}^j orders — Update to b_{it}^j — $t+1$

[12] Some papers relax this assumption, but they either consider a finite-horizon model (e.g., Albornoz, Calvo Pardo, Corcos, and Ornelas (2012)), or one-shot learning (e.g., Nguyen (2012)). Also, they do not estimate a structural model, which is important to quantify the endogenous dynamic relationship between export participation and learning. A direction for future work is to relax this assumption, but assuming the demand factor has the global scope as in Albornoz, Calvo Pardo, Corcos, and Ornelas (2012).

[13] I considered an AR(1) process of productivity evolution, in which there is productivity gain from exporting: $\omega_{it} = g_0 + g_1\omega_{it-1} + g_2 e_{it-1}^j + \epsilon_{it}$, where e_{it-1}^j is an indicator of export decision of last period in a specific market j. It turns out that g_2 is usually insignificant.

where ω_{it} is firm i's productivity in period t, b_{it-1}^j represents the belief about the demand factor in market j at the beginning of period t (or at the end of period $t-1$), and $e_{it}^j = 0$ or 1 represents the export participation in market j in period t.

In what follows, I describe the elements of the model in detail by considering the firm's static decisions (on production and pricing), dynamic decisions (on exporting), as well as the evolution of the firm's belief about foreign demand.

3.2 Static Decisions

The firm's static decision is to set prices for the domestic market and foreign markets (where it decides to export) so that it maximizes its current profit in each period, after observing its productivity, capital stock, and aggregate demand in each market.

3.2.1 Cost of production

Each firm i faces a constant short-run marginal cost to produce its product for domestic and foreign markets. The logarithm of the marginal cost for period t is written as:[14]

$$\ln C_{it} = \gamma_0 + \gamma_w \ln W_{it} + \gamma_k \ln K_{it} - \omega_{it}, \tag{1}$$

where W_{it}, K_{it}, and ω_{it} are the is the wage rate, capital stock, and productivity of firm i in period t respectively. Within each firm and each period, the productivity is the same to produce goods for domestic market as well as all foreign markets.[15] That is, the firm faces the same short-run marginal cost in all markets for the entire period t. However, the marginal cost does vary across firms and over time. The variation is captured by two sources of heterogeneity. The first one is the observable heterogeneity of K_{it} and W_{it}, which is exogenously given in each period.[16] The second

[14]A similar specification is used in Aw, Roberts, and Xu (2011).

[15]I assume the technology employed in manufacturing products for different markets is the same within the firm in each period. Nonetheless, it is possible that firms ship products with different quality (and different marginal costs) to different destinations as documented in Bastos and Silva (2010). In this case, I can use a set of market dummies in the marginal cost function to control for the destination effect. These market dummies can be identified using the difference in average export prices across markets. I exclude the dummies in the specification since I will consider ceramic industry, and there is not much difference in producing product for different markets.

[16]In reality capital stock may evolve endogenously, and more productive firms may choose to invest in the capital and reduce their marginal costs. This leads to firms with high productivity and capital more likely to export. But in

source is captured by ω_{it}, the firm-time specific productivity. It is also observed by the firm, but is not observable by researchers. ω_{it} is assumed to evolve according to an exogenous first-order Markov process:

$$\omega_{it} = g(\omega_{it-1}) + \epsilon_{it}, \tag{2}$$

where ϵ_{it} is an innovation term that is independently drawn from $N(0, \sigma_\epsilon)$.

One important feature of this specification is that the cost-side heterogeneity both across firms and over time is taken into account when I examine the export dynamics with a learning process regarding demand in a foreign market.

3.2.2 Demand and Pricing

The domestic market is assumed to be monopolistically competitive. In particular, firm i faces the Dixit-Stiglitz type demand curve in each period t:

$$Q_{it}^D = Q_t^D \Big(\frac{P_{it}^D}{P_t^D}\Big)^{\eta^D} \equiv \Phi_t^D (P_{it}^D)^{\eta^D}, \tag{3}$$

where Q_t^D and P_t^D are industrial-level output and price index respectively and η^D is the demand elasticity. Note that this demand function describes an aggregate relationship between price and quantity: the total amount of firm i's product sold in the domestic market in the entire period t depends on the industrial aggregate Φ_t^D, firm i's price P_{it}^D, and the constant demand elasticity η^D.

After observing Φ_t^D and its marginal cost C_{it}, the firm sets a price to maximize its profit in the domestic market in period t:

$$\max_{P_{it}^D} \Phi_t^D (P_{it}^D)^{\eta^D} (P_{it}^D - C_{it}). \tag{4}$$

The first order condition implies

$$P_{it}^D = \frac{\eta^D}{1+\eta^D} C_{it}. \tag{5}$$

the empirical application, I consider a relatively short period and the investment in the capital is lumpy, so I follow the literature and treat the capital as exogenous.

Thus the domestic revenue for period t is (in logarithm):

$$\ln R_{it}^D = (\eta^D + 1)\ln(\frac{\eta^D}{\eta^D + 1}) + \ln \Phi_t^D + (\eta^D + 1)(\gamma_0 + \gamma_w \ln W_{it} + \gamma_k \ln K_{it} - \omega_{it}). \tag{6}$$

That is, the domestic revenue can be written as a function of demand elasticity, industrial aggregate, capital stock, and productivity. Notice that although ω_{it} is referred as productivity, it is essentially the combination of physical productivity which influences production cost directly, as well as the effects from product characteristics (e.g., quality) that would affect the quantity of product demanded in all markets.

For each foreign market j, the firm's demand is heterogenous in two aspects. First, firms are different in the expected number of orders they will receive in period t. This reflects differences in the stocks of customers and the sizes of firms' distribution networks. Specifically, if firm i decides to export to market j, then the number of orders it will receive, n_{it}^j, is modeled as an exogenously draw from a truncated Poisson distribution with parameter λ_{it}^j.[17] Note that λ_{it}^j is the expected number of orders in period t. In reality, exporting does not only generate profit but also builds up customer stock over time. To capture this feature, I assume that firms exporting in the last period are likely to receive more orders in the current period. That is, $n_{it}^j \sim \text{Poisson}(\lambda_{it}^j)$, and λ_{it}^j evolves over time according to[18]

$$\ln \lambda_{it}^j = \psi_0 + \psi_1 \ln(\tilde{n}_{it-1}^j) + \psi_2 e_{it-1}^j, \tag{7}$$

where $\tilde{n}_{it-1}^j$ is the number of transactions in the most recent period up to period $t-1$ and e_{t-1}^j is the indicator of export status in period $t-1$.[19] Suppose $\psi_2 > 0$, then this implies that firms continuing to export are likely to have contact with more customers and build up their customer stocks, which will then lead to export growth via increasing in the number of future transactions.[20]

[17] I use the truncated Poisson to model the number of orders because I assume that, if the firm exports to the foreign market, it will receive at least one order.

[18] Arkolakis (2010) provides a theory of market penetration costs in which paying higher costs allows firms to reach an increasing number of consumers in a country. But in this paper I assume this reduce form evolution to keep the analysis tractable.

[19] For example, if the firm exported in period $t-1$ with n_{it-1} transactions, then $\tilde{n}_{it-1} = n_{it-1}$ and $e_{it-1} = 1$. However, if the firm did not export in period $t-1$, but exported in period $t-2$ with n_{it-2} transactions, then $\tilde{n}_{it-1} = n_{it-2}$ and $e_{it-1} = 0$. For firms that never exported, then I use the average number of transactions of exporters as a proxy of $\tilde{n}_{it-1}$ in the empirical estimation.

[20] Note that this evolution process is stationary conditional on the firm's export decision.

Note that λ_{it}^{j} is different across firms and over time but is known by the firm when it makes its export decision.

The second aspect of demand heterogeneity is the quantity demanded at a given price in a single order. With a slight abuse of the notation, I use n as the index of orders (or transactions) of firm i in market j and period t. For each firm i, the quantity sold in each order n in market j is endogenously determined by price P_{in}^{j}, an aggregate demand shifter ϕ_t^j, and an idiosyncratic demand shock which can be decomposed as $\zeta_{in}^{j} = \xi_i^j + u_{in}^{j}$. Here ξ_i^j measures the average demand specific to firm i that is not captured by the aggregate demand shifter ϕ_t^j. It captures all sources of firm i's demand heterogeneity that are unique to market j. For example, it may reflect a combination of customer tastes and the relative product quality difference between the firm and local/other international suppliers in market j. I refer to ξ_i^j as the demand factor, which is a constant over time within firm i and market j but is different across firms and markets. u_{in}^{j} is an unexpected idiosyncratic demand shock associated with order n and is (i.i.d.) drawn from $N(0, \sigma_u^j)$. Specifically, the quantity demanded in transaction n is given by the demand curve:[21]

$$Q_{in}^{j} = \phi_t^j (P_{in}^{j})^{\eta^j} e^{\zeta_{in}^{j}}, \tag{8}$$

where η^j is the demand elasticity in market j.

The firm's profit maximization problem for each specific transaction n is to set a price that maximizes the profit in this transaction. At the beginning of period t, the firm has not observed the demand shock ζ_{in}^{j} yet, but it has observed its marginal cost C_{it} and the aggregate demand shifter ϕ_t^j. Thus, the profit maximization problem for transaction n is

$$\max_{P_{in}^{j}} \phi_t^j (P_{in}^{j})^{\eta^j} (P_{in}^{j} - C_{it}) \mathrm{E}(e^{\zeta_{in}^{j}}). \tag{9}$$

Note that $\mathrm{E}(e^{\zeta_{in}^{j}})$ does not affect the optimal pricing rule, but it does influence the expected quantity

[21] Note that this implies an analogy of *aggregate* expected demand curve as in the domestic market: $E(Q_{it}^{j}) = \lambda_{it}^{j} \phi_t^j (P_{it}^{j})^{\eta^j} E(e^{\xi_i^j}) e^{\sigma_u^2/2} \equiv \Phi_{it}^{j} (P_{it}^{j})^{\eta^j}$, where the firm's heterogeneity in total foreign demand from market j in period t, i.e. Φ_{it}^{j}, consists of λ_{it}^{j} and $E(e^{\xi_i^j})$. A similar specification is employed in Eaton, Eslava, Jinkins, Krizan, and Tybout (2013).

demanded as well as the expected profit in this transaction. Specifically, the first order condition implies the following pricing rule:

$$P_{in}^{j} = \frac{\eta^{j}}{1+\eta^{j}} C_{it}. \tag{10}$$

Thus, the expected profit for a single transaction n is

$$E(\pi_{in}^{j}) = \phi_t^{j} \frac{-1}{1+\eta^{j}} \left[\frac{\eta^{j}}{1+\eta^{j}}\right]^{\eta^{j}} C_{it}^{1+\eta^{j}} E(e^{\zeta_{in}^{j}}). \tag{11}$$

Consequently, the expected total export profit for the entire period t is the sum of profit generated by all transactions in period t:

$$\begin{aligned} E(\Pi_{it}^{j}) &= E\Big(\sum_{n=1}^{n_{it}} \pi_{in}^{j}\Big) = E\Big(\sum_{n=1}^{n_{it}^{j}} \phi_t^{j} \frac{-1}{1+\eta^{j}} \left[\frac{\eta^{j}}{1+\eta^{j}}\right]^{\eta^{j}} C_{it}^{1+\eta^{j}} e^{\zeta_{in}^{j}}\Big) \\ &= \lambda_{it}^{j} \phi_t^{j} \frac{-1}{1+\eta^{j}} \left[\frac{\eta^{j}}{1+\eta^{j}}\right]^{\eta^{j}} C_{it}^{1+\eta^{j}} e^{\sigma_u^2/2} E(e^{\xi_i^{j}}). \end{aligned} \tag{12}$$

The last equation holds because $E(e^{\zeta_{in}^{j}}) = e^{\sigma_u^2/2} E(e^{\xi_i^{j}})$ and the expected number of orders is λ_{it}^{j}. Therefore, the expected total export profit for period t depends on the aggregate demand shifter ϕ_t^{j}, the marginal cost C_{it}^{j}, the expected number of transactions λ_{it}^{j}, and the expectation of the demand factor $E(e^{\xi_i^{j}})$. However, the firm *may* have no previous sales in this market before and thus faces uncertainty about the demand factor ξ_i^{j}. Thus, it is the belief about ξ_i^{j} that determines $E(e^{\xi_i^{j}})$, which in turn influences the expectation of export profit in the entire period t.

Nonetheless, the firm observes the quantity demanded in each order n *after* exporting, which reflects the realization of ζ_{in}^{j}:

$$\zeta_{in}^{j} = \ln Q_{in}^{j} - \ln \phi_t^{j} - \eta^{j} \ln P_{in}^{j}. \tag{13}$$

That is, (Q_{in}^{j}, P_{in}^{j}) contains information about ζ_{in}^{j}. Since ζ_{in}^{j} can be viewed as a signal of ξ_i^{j}, the firm is able to learn about its underlying ξ_i^{j} from these signals and subsequently updates its belief about ξ_i^{j}. As a result, the belief regarding ξ_i^{j} evolves as the firm exports, and this comes to affect the expected value of exporting in the future. The next subsection explains this learning process in greater detail.

3.3 Demand Uncertainty and Bayesian Learning by Exporting

In this section, I characterize uncertainty and Bayesian learning about foreign demand by exporting in a single market j. Since I assume the learning processes of different firms in different markets are independent, I omit the superscript for market j in order to simplify the notation.

I start to model the learning process by specifying firms' knowns and unknowns. The objective of the learning process is the demand factor ξ_i, which is unknown by firm i.[22] However, the firm knows the distribution of ξ for the entire industry in that market, and this serves as a prior belief about its specific ξ_i. More generally, I allow for observable heterogeneity in the prior belief across firms and model it as a function of firm-market characteristics. For example, firms may believe the demand is higher in markets with larger populations; also, some firms may have operated in the foreign market for a long time and have learnt a lot (thus with a low uncertainty) before I observe them in the data set. Formally, each firm holds a (known) prior belief $\xi_i \sim N(m_{i0}, \sigma_{i0})$ at the beginning of the initial period.[23] $m_{i0} = h_m(x_{i0}, z)$ is the prior expectation of the demand factor ξ_i while $\sigma_{i0} = h_\sigma(x_{i0}, z)$ captures the initial uncertainty, where x_{i0} is the firm's characteristics, such as age and ownership, and z is the foreign market's characteristics, such as population and GDP.[24] In particular, if $\sigma_{i0} = 0$, then the firm has no uncertainty about the demand factor.

The firm can learn the true value of the demand factor ξ_i by observing ζ_{in} in each of its transactions. In particular, assuming it is exporting, each firm i observes ζ_{in} as a signal of the demand factor ξ_i. Note that $\zeta_{in} = \xi_i + u_{in}$, but ξ_i and the unexpected idiosyncratic demand shock u_{in} are not separately observed. Thus, the value of ξ_i is not immediately revealed because of the noise u_{in}. However, the firm knows the distribution of the noise: $u_{in} \sim N(0, \sigma_u)$. This knowledge enables the firm to update its belief according to Bayes' rule after observing a series of signals.[25] The standard

[22] Another way to introduce the demand uncertainty is to assume λ_{it} is unknown by the firm. However, the variance of the number of orders is large even within a firm, and this suggests that the learning speed is too slow to be consistent with data. In contrast, modeling the demand factor with uncertainty fits the data better.

[23] Note that "the initial period" means the first period when a firm appears in the data set. It is not necessary to be the period of its first export.

[24] Adding more characteristics, say age, may help to control for the heterogeneity in the prior belief. For example, a firm established 30 years ago may have learnt its demand factor; while a recently founded firm may have never exported to the foreign market before and faces greater uncertainty. In the empirical estimation, to reduce the estimation burden, I will use the first year export status as an initial condition to control for heterogeneity in the prior beliefs, but it is readily be extended to incorporate more firm characteristics.

[25] I can also allow the firm to update the belief after each transaction, but since the firm only makes export decision in the beginning of each period, the two specifications are equivalent.

deviation σ_u determines the speed of learning. In the extreme case where $\sigma_u = 0$, ξ_i can be accurately revealed after just one transaction. However, if σ_u is large, then the firm needs more signals to achieve a given level of accuracy.

More specifically, given the belief at the beginning of period t as $\xi_i \sim N(m_{it-1}, \sigma_{it-1})$, if firm i has decided not to export in period t, then the firm will not observe any new signals of its demand factor and its belief will be the same as it was at the beginning of period $t+1$. However, if the firm has decided to export and conducts n_{it} transactions in period t, then the belief will be updated after receiving n_{it} pieces of information $\{\zeta_{i1}, \zeta_{i2}, \ldots, \zeta_{in_{it}}\}$ about the true demand factor in that period. Consequently, the posterior belief *at the end of period* t is given by $\xi_i \sim N(m_{it}, \sigma_{it})$, where[26]

$$m_{it} = \begin{cases} \dfrac{\sigma_u^2 m_{it-1} + \sigma_{it-1}^2 \tilde{\zeta}_{it}}{\sigma_u^2 + \sigma_{it-1}^2 n_{it}}, & \text{if exported in period } t \\ m_{it-1}, & \text{otherwise} \end{cases} \tag{14}$$

and

$$\sigma_{it}^2 = \begin{cases} \dfrac{\sigma_{it-1}^2 \sigma_u^2}{\sigma_u^2 + \sigma_{it-1}^2 n_{it}}, & \text{if exported in period } t \\ \sigma_{it-1}^2, & \text{otherwise} \end{cases} \tag{15}$$

and

$$\tilde{\zeta}_{it} = \sum_{n=1}^{n_{it}} \zeta_{in}. \tag{16}$$

Alternatively, the above equations can be written in terms of the initial belief and the entire history of signals received until period t: $\{\zeta_{i1}, \zeta_{i2}, \ldots, \zeta_{iN_{it}}\}$ (where $N_{it} = \sum_{\tau=1}^{t} n_{i\tau}$):

$$m_{it} = \frac{\sigma_u^2 m_{i0} + \sigma_{i0}^2 \overline{\zeta}_{it}}{\sigma_u^2 + N_{it}\sigma_{i0}^2}, \tag{17}$$

and

$$\sigma_{it}^2 = \frac{\sigma_{i0}^2 \sigma_u^2}{\sigma_u^2 + N_{it}\sigma_{i0}^2}, \tag{18}$$

[26]See DeGroot (2005), Chapter 9.

and

$$\bar{\zeta}_{it} = \sum_{n=1}^{N_{it}} \zeta_{in}. \tag{19}$$

Note that, in each period t, the belief about ξ_i is characterized by two variables: the mean and the standard deviation of the belief. The mean represents the expectation of the demand factor ξ_i and may fluctuate over time depending on the entire history of signals that the firm received. However, the standard deviation, which measures the magnitude of uncertainty, is strictly decreasing as the firm receives more signals. As a result, for some firms the expectation of the belief will increase if the firms keep observing large quantities purchased at given prices, but for others the expectation may decrease if their transaction quantities are at lower levels than expected. For all firms, however, standard deviations will keep falling if the firms keep exporting, which implies that uncertainty is decreasing over time as the firms receive more signals.

3.4 Dynamic Decision – Export Participation with Learning about demand

In this section, I characterize a forward-looking firm's export participation with learning about the demand factor in a single market j. Again, I omit the superscript for market j to simplify the notation. The term "export" or "not to export" means "export" or "not to export" to a *specific* market j. *Within* market j, the export decision and the learning process are endogenously related. The firm's export decision depends on its current belief regarding the demand factor; moreover if the firm decides to export, then it will expect the belief in the next period to be updated according to the signals received from exporting. Hence, the export decision is dynamic not only because of the sunk entry cost that the firm has to pay if it did not export in the last period but also because of this endogenous learning process. However, the assessment of uncertainty and the learning process will be biased if the persistence introduced by the sunk entry cost is ignored. Thus, following the literature (e.g., Roberts and Tybout (1997); Das, Roberts, and Tybout (2007); Aw, Roberts, and Xu (2011)), I take the effect of the sunk entry cost into account by assuming: if the firm decides to export but it did not export in the last period, it must pay a sunk cost c^s_{it}; otherwise, it pays a fixed cost c^f_{it}. I assume that c^f_{it} and c^s_{it} are independently drawn from distributions $G_f(\cdot)$ and $G_s(\cdot)$, respectively.

At the beginning of period t, given the current belief $N(m_{it-1}, \sigma_{it-1})$ regarding the demand factor, the expected total export profit in period t (before considering the fixed/sunk cost) is the sum of profit from all transactions in period t:

$$
\begin{aligned}
E\Big[\Pi(s_{it}, e_{it-1})\Big] &= E\Big(\sum_{n=1}^{n_{it}} \pi(s_{it})\Big) = E\Big(\sum_{n=1}^{n_{it}} \phi_t \frac{-1}{1+\eta}\Big[\frac{\eta}{1+\eta}\Big]^{\eta} C_{it}^{1+\eta} e^{\zeta_{in}}\Big) \\
&= \lambda_{it}\phi_t \frac{-1}{1+\eta}\Big[\frac{\eta}{1+\eta}\Big]^{\eta} C_{it}^{1+\eta} \exp(m_{it-1} + \sigma_{it-1}^2/2 + \sigma_u^2/2), \\
&= \lambda_{it}\tilde{\phi}_{it} \exp(m_{it-1} + \sigma_{it-1}^2/2 + \sigma_u^2/2 - (\eta+1)\omega_{it}),
\end{aligned}
\tag{20}
$$

where $\lambda_{it} = \exp(\psi_0 + \psi_1 \log(\tilde{n}_{it-1}) + \psi_2 e_{it-1})$, $\tilde{\phi}_{it} = \phi_t \frac{-1}{1+\eta}\left[\frac{\eta}{1+\eta}\right]^{\eta} e^{\gamma_0(1+\eta)} W_{it}^{\gamma_w(1+\eta)} K_{it}^{\gamma_k(1+\eta)}$, and e_{it-1} is the dummy variable indicating the export status in period $t-1$. The set of state variables is summarized in $s_{it} = (\tilde{\phi}_{it}, \tilde{n}_{it-1}, \omega_{it}, m_{it-1}, \sigma_{it-1})$.[27]

Thus, the expected total export profit in period t depends on the expected number of orders, productivity, the current belief about the demand factor, and an aggregate demand/cost shifter $\tilde{\phi}_{it}$. In particular, if the firm exported in the last period, then it will expect an increase in the number of orders that it will receive in this period. This implies an increase in both the total export volume and profit for firm i in period t.

Also, the current belief characterized by $(m_{it-1}, \sigma_{it-1})$ is a part of the state variables, since it affects the expected profit. More specifically, the expected profit is increasing in both the mean m_{it-1} and the standard deviation σ_{it-1} of the current belief, holding other variables fixed. This implies that, if the firm has a higher expectation of the demand factor then it expects more profit in this period; also, if the firm faces greater uncertainty about the demand factor then it is more likely to export because of the option value of learning. This feature is a result of the assumption that profit is an increasing and convex function of the demand shock ζ_{it}, which is commonly assumed in the literature and also employed in this model.

The timing of the entire model is summarized as follows:

1. At the beginning of period t, the firm observes (s_{it}, e_{it-1}), where e_{it-1} is a dummy variable indicating whether the firm exported or not in period $t-1$;

[27] Note that e_{it-1} is also a part of the state variables, but I pull it out from s_{it} to simplify the later notations.

2. After observing its fixed cost draw c_{it}^f or sunk cost draw c_{it}^s, the firm decides whether to export or not (i.e., choose $e_{it} = 0$ or 1), based on its current state (s_{it}, e_{it-1}) which includes its current productivity and belief about the demand factor characterized by $(m_{it-1}, \sigma_{it-1})$;

3. If the firm decides to export, it pays the fixed cost c_{it}^f if it exported in the last period or pays the sunk cost c_{it}^s otherwise. During period t, the firm receives and fulfills n_{it} orders from customers in the foreign market. The quantity and the price of each order is determined by equations 8 and 10 respectively. The pricing decision is static. The firm is able to observe a signal ζ_{in} about ξ_i in each order after exporting. At the end of this period, a series of signals $\{\zeta_{i1}, \zeta_{i2}, \ldots, \zeta_{in_{it}}\}$ is observed, which is then used to update its belief about ξ_i. The posterior belief is (m_{it}, σ_{it}), according to equations 14 and 15;

4. If the firm decides not to export, then there is no export profit from this market in this period; there is also no update to the belief at the end of this period (i.e., $m_{it} = m_{it-1}$ and $\sigma_{it} = \sigma_{it-1}$);

5. Period $t + 1$ begins and all other state variables are updated. In particular, its productivity evolves to ω_{it+1}.

Given the timing, I model the firm's dynamic export participation using a Bellman equation. I denote the expected value function at the beginning of each period t before observing the fixed cost draw or the sunk cost draw as $V_t(s_{it}, e_{it-1})$. The firm will choose to export to this market if the expected total (current plus future) payoff is greater than the cost (i.e., fixed cost or sunk cost) it must pay. Thus, the Bellman equation is given by

$$
\begin{aligned}
& V_t(s_{it}, e_{it-1}) \\
&= E_{c^f, c^s} \max \begin{cases} \delta E[V_{t+1}(s_{it+1}, 0) | s_{it}, e_{it-1}], & \text{if} \quad e_{it} = 0 \\ E[\Pi(s_{it}, e_{it-1})] - e_{it-1} c_{it}^f - (1 - e_{it-1}) c_{it}^s + \delta E[V_{t+1}(s_{it+1}, 1) | s_{it}, e_{it-1}], & \text{if} \quad e_{it} = 1 \end{cases}
\end{aligned} \tag{21}
$$

where δ is the discount rate, and

$$\begin{aligned}
&E[V_{t+1}(s_{it+1}, e_{it})|s_{it}, e_{it-1}] \\
&= \int V_{t+1}(s_{it+1}, e_{it})dF(s_{it+1}|s_{it}, e_{it-1}, e_{it}) \\
&= \int V_{t+1}(s_{it+1}, e_{it})dF_{\omega}(\omega_{it+1}|\omega_{it})dF_b(m_{it}, \sigma_{it}, \tilde{n}_t|s_{it}, e_{it-1}, , e_{it}),
\end{aligned} \tag{22}$$

and $F_{\omega}(\omega_{it+1}|\omega_{it})$ and $F_b(m_{it}, \sigma_{it}, \tilde{n}_t|s_{it}, e_{it-1}, e_{it})$ are the transition probabilities of the four key state variables (productivity, the mean and the standard deviation of the belief, and the number of orders in the most recent period). It is important to note that the transition of productivity is independent from the export decision. However, the transition of the belief about the demand factor is affected by the export decision as well as the number of orders actually received in period t. Also, there is no direct correlation between the two transition probabilities, $F_{\omega}(\cdot|\cdot)$ and $F_b(\cdot|\cdot)$. I now turn to specify the transition probabilities of all four key state variables.[28]

First, $F_{\omega}(\omega_{it+1}|\omega_{it})$ is the distribution of productivity in period $t+1$, given the productivity in period t. Specifically, given productivity evolution in equation 2, ω_{it+1} is drawn from $N(g(\omega_{it}), \sigma_{\epsilon})$. That is,

$$F_{\omega}(\omega_{it+1}|\omega_{it}) = N(g(\omega_{it}), \sigma_{\epsilon}). \tag{23}$$

Second, $F_b(m_{it}, \sigma_{it}, \tilde{n}_t|s_{it}, e_{it-1}, e_{it})$ is the joint distribution of the belief and $\tilde{n}_t$ at the beginning of period $t+1$, given the current state s_{it} and the export decision (e_{it-1}, e_{it}). Note that the distribution of the belief and $\tilde{n}_t$ are not independent, since the updated belief is related to the number of received signals which are contained in orders. Specifically, this joint probability is determined according to equations 14 and 15, and I explain it in detail as follows.

Given the belief at the beginning of period t as

$$\xi_i \sim N(m_{it-1}, \sigma_{it-1}), \tag{24}$$

then the posterior belief at the end of period t (or the beginning of period $t+1$) remains the same

[28]Since the other state variable $\tilde{\phi}_t$ is an aggregate index of capital stock and the year dummy, I following Aw, Roberts, and Xu (2011) to assume the firm forms a rational perception of the sequence of $\tilde{\phi}_t$.

if the firm has decided not to export in period t. That is, the transition probability is degenerate:

$$F_b(m_{it} = m_{it-1}, \sigma_{it} = \sigma_{it-1}, \tilde{n}_{it} = \tilde{n}_{it-1} | s_{it}, e_{it-1}, e_{it} = 0) = 1.$$

On the other hand, if the firm has decided to export, and suppose for the entire period t it receives a total of n_{it} signals $\{\zeta_{i1}, \zeta_{i2}, \ldots, \zeta_{in_{it}}\}$, then the updated belief is

$$\xi_i \sim N(m_{it}, \sigma_{it}), \tag{25}$$

where

$$m_{it} = m_{it-1} + n_{it} \frac{\sigma_{it}^2}{\sigma_u^2} \Big(\frac{1}{n_{it}} \sum_{n=1}^{n_{it}} (\xi_i + u_{in}) - m_{it-1} \Big) \tag{26}$$

and

$$\sigma_{it}^2 = \frac{\sigma_{it-1}^2 \sigma_u^2}{n_{it} \sigma_{it-1}^2 + \sigma_u^2}. \tag{27}$$

Since m_{it-1} and σ_{it-1} are known at period t, the transition depends on random variables (n_{it}, ξ_i, u_{in}). Note that the distributions of n_{it} and u_{in} are known, and ξ_i is believed to be distributed as the current belief of firm i (i.e. $N(m_{it-1}, \sigma_{it-1})$). Thus, *conditional on* n_{it}, the distribution of m_{it} is

$$m_{it} \sim N\Big(m_{it-1}, n_{it} \frac{\sigma_{it}^2}{\sigma_u^2} \sqrt{\sigma_u^2 / n_{it} + \sigma_{it-1}^2}\Big) \equiv F_m(m_{it} | n_{it}, m_{it-1}, \sigma_{it-1}, e_{it} = 1), \tag{28}$$

and the distribution of σ_{it} is degenerate:

$$F_\sigma(\sigma_{it} = \frac{\sigma_{it-1}^2 \sigma_u^2}{n_{it} \sigma_{it-1}^2 + \sigma_u^2} | n_{it}, m_{it-1}, \sigma_{it-1}, e_{it} = 1) = 1. \tag{29}$$

Since n_{it} is drawn from the truncated Poisson distribution with known parameter $\lambda_{it} = \exp(\psi_0 + \psi_1 \log(\tilde{n}_{it-1}) + \psi_2 e_{it-1})$,[29] the probability to receive n_{it} orders in period t is

$$F_n(n_{it} | s_{it}, e_{it-1}) = \frac{\lambda_{it}^{n_{it}} e^{-\lambda_{it}}}{n_{it}! (1 - e^{-\lambda_{it}})}. \tag{30}$$

[29] Note that λ_{it} is known given the state (s_{it}, e_{it-1}).

Therefore, the joint transition probability of $(m_{it}, \sigma_{it}, \tilde{n}_{it})$ is given by

$$\begin{aligned} & F_b(m_{it}, \sigma_{it}, \tilde{n}_{it} | s_{it}, e_{it-1}, e_{it}) \\ & = F_m(m_{it} | n_{it}, s_{it}, e_{it} = 1) F_\sigma(\sigma_{it} | n_{it}, s_{it}, e_{it} = 1) F_n(n_{it} | s_{it}, e_{it-1}) \end{aligned} \tag{31}$$

Thus, if the firm decides not to export in period t, then there is no transition in the belief. But if the firm decides to export, then the joint transition probability of the belief consists of the distributions of m_{it} and σ_{it}, conditional on the realization of n_{it} and the current state of the belief $(m_{it-1}, \sigma_{it-1})$.

In this way, I incorporate heterogeneity in both productivity and the demand belief into a model of exporting. The firm's export decision depends on both productivity evolution and the evolution of the demand belief. In particular, productivity evolves exogenously and influences the export decision by affecting the marginal cost of production. The demand belief evolves endogenously, depending on the export decision and export outcomes. The next section demonstrates the strategy of identifying the effects of the two processes in determining export dynamics and estimating the structural model.

4 Identification and Estimation Strategy

There are two major sources driving the export dynamics in this model. The first one is the evolution of productivity, and the second one is the evolution of the belief regarding the demand factor. Both sources are heterogeneous across firms and over time, and neither of them is observable to researchers. I rely on two sets of data to identify the role of each source. The first data set provides firm-level production information, which includes employment, labor and material expenditures, capital stock, and domestic revenue for each firm in each period. The second data set contains firm shipment-level exports, including the export destination, quantity and price of each shipment, and shipment month. The strategy for identification is to utilize the fact that productivity affects both domestic revenue and export participation while the demand belief only influences export participation.

To be specific, domestic revenue depends on current productivity; it does not depend on the evolution of the belief in any of the foreign markets. In particular, high productivity implies a low marginal cost, which allows the firm to set a low price and increases domestic revenue. Thus, the relationship between domestic revenue and productivity enables me to recover the productivity for each firm in each period. In turn, with the recovered firm-level productivity, I am able to estimate productivity evolution before considering the dynamic export decision.[30]

The observed export participation in each foreign market, together with shipment-level prices and quantities, allows me to recover the demand belief of each firm in each period (up to a set of parameters to be estimated) and to identify the learning process in each foreign market. Specifically, the price and quantity sold in each shipment imply a demand signal received by the firm, which is then used to recover the expectation of the belief. The number of shipments in each period allows me to recover the magnitude of uncertainty of the belief. Export participation depends on both the expectation and uncertainty of the belief, but in different ways. The probability of exporting is increasing in the expectation, which fluctuates over time depending on the value of the demand signals received. However, since the option value of learning is decreasing when uncertainty is resolved over time, the export probability is decreasing in the number of received signals in a deterministic style, holding other factors fixed. In particular, a model with only productivity heterogeneity predicts more productive firms export. While in my model with the two-dimensional heterogeneity, firms face more uncertainty about demand may also export because of the large option value of learning even if their productivity is not high enough. Thus, with both productivity and demand beliefs being recovered, the cross-sectional and time series patterns of export decisions identify the role of each driving force.

The estimation approach is inspired by the identification strategy. I divide the full set of parameters into a set of static parameters and a set of dynamic parameters. I first estimate the static parameters, time-varying productivity, and demand signals received by each firm. Then I estimate the dynamic parameters.

[30] It is possible to allow productivity gain from exporting to a *single* foreign market, similar to Aw, Roberts, and Xu (2011). This will not break the identification of productivity from domestic sales, but will make the analysis of export participation complicated. Since no significant productivity gain found in the industry I am going to study, I assume there is no such "learning by doing" in exporting for simplification.

4.1 Estimation of Static Parameters, Productivity, and Demand Signals

As the first step, I estimate the set of static parameters: demand elasticity in the domestic market (η^D) and each foreign market (η^j), and the marginal cost parameters. In addition, for each firm, I recover its time-varying productivity and demand signals received in each market and each period. I use firm-level production information to estimate the marginal cost function and to recover productivity. Then I aggregate shipment-level exports into firm-market-level exports, and estimate the demand elasticities from the relationship between total variable costs and firm-market-level exports. Finally, I recover the market-level demand signals for each firm using shipment-level exports. These demand signals will be used to update firms demand beliefs in the dynamic estimation stage. The implementation of this strategy is specified as follows.

First, I estimate the marginal cost parameters and productivity using the firm-level production and domestic sale information. Specifically, the domestic revenue function (6) implies

$$\ln R_{it}^D = (\eta^D + 1)\ln(\frac{\eta^D}{\eta^D + 1}) + \ln \Phi_t^D + (\eta^D + 1)(\gamma_0^D + \gamma_w \ln W_{it} + \gamma_k \ln K_{it} - \omega_{it}) + v_{it}, \tag{32}$$

where v_{it} is the measurement error. Note that the firm's productivity can be correlated with its capital stock. Thus, to control for the unobservable productivity ω_{it}, I follow Olley and Pakes (1996) and Levinsohn and Petrin (2003) to rewrite the unobserved productivity in terms of related observable variables. In particular, firms' choice of variable material and labor inputs, M_{it} and L_{it}, depends on the level of productivity and the demand beliefs about export markets. Since I assume that marginal cost is constant in output, the relative input will not be a function of total output and thus not depend on demand beliefs about export markets. Also, if technology differences are not Hick's neutral, then the differences in the mix of the two inputs across firms and over time will reflect differences in productivity level.[31] Thus, I can write the unobserved productivity as a function of the relative input, conditional on the capital stock level: $\omega_{it} = \omega(K_{it}, M_{it}/L_{it})$. Then I combine the demand elasticity terms into an intercept $\tilde{\gamma}_0^D$ and use a set of time dummies, $\tilde{\Phi}_t$, to

[31]Non-Hicks neutral productivity has been found in a large empirical literature. See Stevenson (1980) for a model using plant-level data. Aw, Roberts, and Xu (2011) also utilize the same idea to recover productivity.

capture the domestic industrial aggregate Φ_t^D. Thus, the above equation can be written as:

$$\ln R_{it}^D = \tilde{\gamma}_0^D + \sum_{t=1}^{T} \gamma_t \tilde{\Phi}_t + \gamma_w \ln W_{it} + f(K_{it}, M_{it}/L_{it}) + v_{it}, \tag{33}$$

where $f(K_{it}, M_{it}/L_{it}) = (1+\eta^D)(\gamma_K \ln K_{it} - \omega(K_{it}, M_{it}/L_{it}))$ is a function of capital stock, material input and labor input. I parameterize function f as a cubic polynomial of its arguments. Now the error term v_{it} is uncorrelated with the right-hand-side variables. Thus, I use ordinary least square regression to obtain the estimates. An important output from the regression is the fitted value of function f, which is denoted as $\hat{f}_{it}$. This is an estimate of $(1+\eta^D)(\gamma_K \ln K_{it} - \omega_{it})$. That is,

$$\hat{f}_{it} = (1+\eta^D)(\gamma_K \ln K_{it} - \omega_{it}). \tag{34}$$

Then, I follow Olley and Pakes (1996) to construct a series of productivity measures for each firm, by utilizing the productivity evolution process. In particular, the first-order Markov process of productivity evolution is specified as

$$\omega_{it} = g_0 + g_1 \omega_{it-1} + \epsilon_{it}. \tag{35}$$

Substitute $\omega_{it} = -\frac{1}{\eta^D+1}\hat{f}_{it} + \gamma_K \ln K_{it}$ into the above evolution process and get

$$\hat{f}_{it} = -(\eta^D+1)g_0 + g_1 \hat{f}_{it-1} + (\eta^D+1)\gamma_K \ln K_{it} - g_1(\eta^D+1)\gamma_K \ln K_{it-1} - (\eta^D+1)\epsilon_{it}. \tag{36}$$

Again, the error term is uncorrelated with all right-hand-side variables. Thus, this equation can be estimated by nonlinear least squares, since the function is nonlinear in g_1 and $(\eta^D+1)\gamma_K$. The key parameters estimated in this equation are $g_0^* = (\eta^D+1)g_0$, g_1, and $\gamma_K^* = (\eta^D+1)\gamma_K$. Note that η^D, g_0, and γ_K are not separately identified. However, if η^D is known, then $g_0 = \frac{g_0^*}{\eta^D+1}$ and $\gamma_K = \frac{\gamma_K^*}{\eta^D+1}$ are immediately recovered. More importantly, from equation 34, I can recover productivity as $\omega_{it} = -\frac{1}{\eta^D+1}\hat{f}_{it} + \gamma_K \ln K_{it}$ with knowledge of η^D.

To estimate η^D, I follow Aw, Roberts, and Xu (2011) and utilize the relationship between the total variable cost (TVC) and domestic revenue (R^D) as well as the total export revenue in each market (X^j). Since the marginal cost of production is the same for domestic sales and exports to all markets, the first order conditions for profit maximization of the domestic and foreign markets imply that the total variable cost is an elasticity-weighted combination of total revenue in each market. Specifically, for each firm i and each period t:

$$\begin{aligned} TVC_{it} &= Q_{it}^D C_{it} + \sum_{j=1}^{J} Q_{it}^j C_{it} \\ &= (1+\frac{1}{\eta^D})Q_{it}^D P_{it}^D + \sum_{j=1}^{J}(1+\frac{1}{\eta^j})Q_{it}^j P_{it}^j \\ &= (1+\frac{1}{\eta^D})R_{it}^D + \sum_{j=1}^{J}(1+\frac{1}{\eta^j})X_{it}^j, \end{aligned} \tag{37}$$

where Q_{it}^j is the total quantity exported to market j by firm i in period t, X_{it}^j is the corresponding total revenue, and R_{it}^D is the total revenue in the domestic market. Note that although the firm may export to market j with multiple transactions, the demand function (i.e., equation 8) implies that the optimal price is proportional to the marginal cost, as shown in equation 10. Thus the second equality in the above equation holds. Therefore, the following empirical equation can be used to estimate η^D as well as all η^js:

$$TVC_{it} = (1+\frac{1}{\eta^D})R_{it}^D + \sum_{j=1}^{J}(1+\frac{1}{\eta^j})X_{it}^j + \upsilon_{it}, \tag{38}$$

where υ_{it} is the measurement error.

Hence, up to now I have obtained the key estimates $\hat{g}_0$, $\hat{g}_1$, $\hat{\gamma}_K$, $\hat{\eta}^D$, and all $\hat{\eta}^j$s. Also, it is straightforward to recover the productivity for each firm in each period as $\omega_{it} = -\frac{1}{\hat{\eta}^D+1}\hat{f}_{it} + \hat{\gamma}_K \ln K_{it}$.

In addition, with the estimates of all $\hat{\eta}^j$s, I can recover the aggregate demand shifter ϕ_t^j in the foreign demand function (i.e., equation 8) using shipment-level exports via ordinary least squares. Specifically, the logarithm of the demand function in foreign market j is

$$\ln Q_{in}^j = \ln \phi_t^j + \hat{\eta}^j \ln P_{in}^j + \zeta_{in}^j, \tag{39}$$

where $\zeta_{in}^j = \xi_i^j + u_{in}^j$ is the error term with demand factor ξ_i^j. Since the demand factor is uncorrelated with the aggregate demand shifter, the regression produces unbiased estimates of ϕ_t^j.[32] The regression also provides an estimate of the standard deviation of the noise by using the demeaned version of equation 39 to eliminate the unobservable firm-fixed effect ξ_i^j:[33]

$$\hat{\sigma}_u^j = \sqrt{\frac{1}{\sum_i N_i^j} \sum_{i,n} (\Delta \ln Q_{in}^j - \Delta \ln \hat{\phi}_t^j - \hat{\eta}^j \Delta \ln P_{in}^j)^2}, \tag{40}$$

where N_i^j is the total number of transactions from firm i to market j, $\Delta \ln Q_{in}^j = \ln Q_{in}^j - \frac{1}{N_i^j} \sum_n \ln Q_{in}^j$, $\Delta \ln P_{in}^j = \ln P_{in}^j - \frac{1}{N_i^j} \sum_n \ln P_{in}^j$, and $\Delta \ln \hat{\phi}_t^j = \ln \hat{\phi}_t^j - \frac{1}{N_i^j} \sum_n \ln \hat{\phi}_t^j$.

After obtaining these estimates, I can recover the demand signals received by firms as

$$\zeta_{in}^j = \ln Q_{in}^j - \ln \hat{\phi}_t^j - \hat{\eta}^j \ln P_{in}^j. \tag{41}$$

Then I use demand signals to update firms' market-specific demand beliefs according to Bayes' rule as specified in equations 17 and 18.[34]

To sum up, after the static estimation, three major objectives have been achieved. First, I have estimated the parameters in the marginal cost function. Second, I have obtained a productivity measure for each firm in each period, and the productivity evolution process has been recovered. Third, I have recovered the market-specific demand signals received by each firm in each period. These signals enable me to write firms' demand beliefs as a function of initial beliefs up to a set of parameters to be estimated in the dynamic stage.

4.2 Estimation of Dynamic Parameters

The set of dynamic parameters includes the parameterized initial belief function $h_m(\cdot|\beta_m)$, $h_\sigma(\cdot|\beta_\sigma)$, distributions of fixed cost $G_f(\cdot|\beta_f)$ and sunk cost $G_s(\cdot|\beta_s)$, and the parameters for Poisson parameter

[32] To avoid potential endogeneity problem, I use a balanced data in this estimation. Also, the time dummies and the firm demand factor is not separately identified. I normalize the first year dummy as 10, but this does not affect the dynamic estimation stage.

[33] The demeaned version of equation 39 implies $u_{it}^j = \Delta \ln Q_{in}^j - \Delta \ln \hat{\phi}_t^j - \hat{\eta}^j \Delta \ln P_{in}^j + \overline{u}_i^j$, where $\overline{u}_i^j = \frac{1}{N_i^j} \sum_n u_{in}^j$.

[34] Notice that the year dummies and demand factors are not separately identified. Without loss of generality, I normalize the first year dummy as zero.

evolution ψ_0, ψ_1, and ψ_2 in equation 7, where $\beta_m, \beta_\sigma, \beta_f, \beta_s$ are vectors of parameters associated with the corresponding functions.

4.2.1 Estimation Details

I estimate the dynamic parameters via the Maximum Likelihood Method. The likelihood is constructed from data on the discrete choice of export participation together with the number of transactions for each firm in each period and each market. To simplify notation, I only consider the exports to a single market j and omit the market superscript, since the export decisions and learning processes in different markets are independent for each firm. However, it is straightforward to extend the estimation to all foreign markets. In particular, for each firm i in each period t, I observe the export participation dummy e_{it}. If the firm exported in period t, then $e_{it} = 1$, and I can observe the total number of transactions n_{it}, as well as the price (P_{in}) and quantity (Q_{in}) in each transaction n.[35] If the firm did not export to that market in period t, then $e_{it} = 0$ and no transaction happened.

The full likelihood consists of two partial likelihoods. The first one is about the number of transactions n_{it} of each firm i in each period t. The parameters involved include the parameters for the evolution process of λ_{it}, which is summarized in the vector $\theta_1 = (\psi_0, \psi_1, \psi_2)$. The first partial likelihood is given by the truncated Poisson probability since the number of transactions is assumed to be greater than zero if the firm exports:

$$\ell^1(n_{it}; \theta_1) = \frac{\lambda_{it}^{n_{it}} e^{-\lambda_{it}}}{n_{it}!(1 - e^{-\lambda_{it}})}, \tag{42}$$

where λ_{it} is specified as equation 7.

Given θ_1, the second partial likelihood is about the discrete choice of export participation e_{it}, conditional on (s_{it}, e_{it-1}). The parameters involved are parameters for initial beliefs and distributions of fixed and sunk entry costs, which are summarized in the vector $\theta_2 = (\beta_m, \beta_\sigma, \beta_f, \beta_s)$. The

[35] Note the superscript of market j is suppressed, and I use n as an index of transactions.

second partial likelihood is given by:

$$\begin{aligned} &\ell^2(e_{it};\theta_1,\theta_2) \\ &= e_{it}\Pr(e_{it}=1|s_{it},e_{it-1};\theta_1,\theta_2) + (1-e_{it})\Pr(e_{it}=0|s_{it},e_{it-1};\theta_1,\theta_2) \end{aligned} \tag{43}$$

where $\Pr(e_{it} = 1|s_{it}, e_{it-1}; \theta_1, \theta_2)$ is the conditional probability of exporting. It depends on the parameters θ_1 and θ_2. Given the parameterized distributions of fixed cost and sunk cost, the conditional probability of exporting is

$$\begin{aligned} &\Pr(e_{it}=1|s_{it},e_{it-1},\theta_1,\theta_2) \\ &= \Pr\Big(E[\Pi(s_{it},e_{it-1})] + \delta E[V(s_{it+1},1)|s_{it},e_{it-1}] - \delta E[V(s_{it+1},0)|s_{it},e_{it-1}] \\ &> e_{it-1}c^f + (1-e_{it-1})c^s]\Big), \end{aligned} \tag{44}$$

where δ is the discount factor, $E[\Pi(s_{it}, e_{it-1})]$ is the period export profit implied by equation 20, and $E[V(s_{it+1}, e_{it})|s_{it}, e_{it-1}]$ is the expected value of exporting at the beginning of period $t+1$, as described in the Bellman equation. Note that the belief affects this probability through both the expected period export revenue $E[\Pi(s_{it}, e_{it-1})]$ and the expected future value of exporting $E[V(s_{it+1}, e_{it})|s_{it}, e_{it-1}]$.

Thus, the full likelihood is the product of the two partial likelihoods:

$$\ell^f(e_{it},n_{it};\theta_1,\theta_2) = \ell^1(n_{it}|\theta_1)\ell^2(e_{it}|\theta_1,\theta_2). \tag{45}$$

I denote the full set of dynamic parameters as $\theta = (\theta_1, \theta_2)$. Then, θ can be estimated via maximizing the likelihood:

$$\hat{\theta} = \arg\max_{\theta} \sum_{i,t} \log(\ell^f(e_{it},n_{it};\theta_1,\theta_2)). \tag{46}$$

It is computationally difficult to obtain an estimate of θ in this way. The evaluation of the full likelihood requires solving for the value function. Given the high dimension of the state variables, this is a major computational burden in the estimation. Also, θ_1 increases the dimension of parameters to be estimated. This in turn makes the estimation even more time-consuming. To solve this issue, I reduce the computational burden in two directions. First, I adopt the strategy proposed by Rust (1987) to estimate the parameters by three stages, which is described in the remainder of this subsection. Second, I follow Nagypál (2007) to compute the value function, which is specified in the next subsection in detail.

Like Rust (1987), I estimate the parameters in three stages. The first stage estimates θ_1 via the first partial likelihood. In particular, I estimate θ_1 using the data on the number of transactions and export participation of each firm in each period. To be specific, θ_1 is obtained as

$$\hat{\theta}_1 = \arg\max_{\theta_1} \sum_{i,t} \log(\ell^1(n_{it}; \theta_1)). \tag{47}$$

Note that in this estimation, there is no need to calculate the value function. In this way, the first stage provides an estimate of $\theta_1 = (\psi_0, \psi_1, \psi_2)$.

Then, with the estimated $\hat{\theta}_1$, the second stage is to estimate θ_2 via the second partial likelihood:

$$\hat{\theta}_2 = \arg\max_{\theta_2} \sum_{i,t} \log(\ell^2(e_{it}; \hat{\theta}_1, \theta_2)). \tag{48}$$

This stage requires the internal computation of the value function at each evaluation of ℓ^2, according to the Nested Fixed Point Algorithm. The detailed algorithm of value function computation will be described in the next subsection. However, the obvious advantage of this estimation is that there are fewer parameters to be estimated, which reduces the estimation burden.

The third stage is to use the estimated $(\hat{\theta}_1, \hat{\theta}_2)$ as an initial starting value to produce an efficient estimate of θ via the full likelihood:

$$\hat{\theta} = \arg\max_{\theta} \sum_{i,t} \log(\ell^f(e_{it}, n_{it}; \theta_1, \theta_2)). \tag{49}$$

This stage also involves the internal calculation of the value function for each evaluation of ℓ^f. Drawing on the work of Rust (1987), this estimation yields a consistent estimator of asymptotic covariance matrix for θ. Nonetheless, the estimate of θ from this stage is usually identical to the estimates from the first two stages.

Since the above estimations require the internal calculation of the value function in each evaluation of the likelihood, it is important to compute it efficiently. I now turn to the algorithm of value function computation in greater detail.

4.2.2 Value Function Computation

Now I describe the algorithm used to solve for the value function, given a set of parameters θ.

First, I construct the demand belief for each firm in each period using the demand signals recovered in equation 41. I consider the belief as a two-dimensional state variable in the value function. Specifically, the full set of state variables includes: an aggregate state $\tilde{\phi}_{it}$, the number of transactions in the most recent period $\tilde{n}_{it-1}$, productivity ω_{it}, belief state m_{it-1} and σ_{it-1}. As a result, the curse of dimensionality makes it difficult to follow the traditional value function iteration method to derive the underlying value function. To solve this problem, I follow the method implemented in Nagypál (2007) to compute the value function. This method utilizes the fact that the magnitude of uncertainty (measured by the standard deviation σ_{it} of the belief) is strictly decreasing in the number of transactions (signals). Thus, at the limit state, the firm will have no uncertainty about the demand factor and $\sigma_{it} = 0$. The value function at this limit state has one fewer state variable. More importantly, the transition of the belief state is degenerate: $\Pr(m_{it} = m_{it-1} = \xi_i) = 1$ and $\Pr(\sigma_{it} = 0) = 1$. These features significantly simplify the computation of the value function at the limit state. Once this is computed, I approximate the firm's value function after observing a large number of signals (so that σ_{it} is small enough, e.g., $\sigma^* = 0.01$) as the value function at the limit state. Using this approximated value function with uncertainty σ^*, I calculate the value function with arbitrarily greater uncertainty (any larger σ_{it}) via backward induction. The details of this procedure are explained in Appendix A.

5 Data

I will use the model developed above to study how the two unobservable driving processes separately explain the firm's export participation. To do this, I utilize two major pieces of information to identify productivity evolution and Bayesian learning about foreign demand from each other. The first one is the firm-level input and output production information, and the second one is the shipment-level exports. Thus, I draw data from two sources.

The first source is the Chinese Monthly Customs Transactions data set. This data set includes all export shipments of Chinese firms from 2000 to 2006. It is common that firms have multiple shipments to a single destination market within a year. Across all industries, the average number of shipments in a year is 8.4 and the standard deviation is 31. Each shipment contains shipment value, quantity, 8-digit HS code, type of trade, destination market, shipment month, and firm identification number. Such detailed information makes it possible to analyze how firms' export participation is endogenously correlated to Bayesian learning about demand. In particular, this distinguishes the export decisions and the export transactions in different destination markets and allows me to recover the demand signals received by firms from the shipment-level quantities and unit prices. Comparing commonly used annual export data sets, this enables me to track the change of export patterns in each specific market over time, and in turn to attribute such dynamic patterns separately to heterogeneity in productivity and Bayesian learning.

One important feature of exports from Chinese manufacturing firms is that a significant portion of transactions is processing trade, in which domestic firms' intermediate material and even related technology are directly supplied by foreign firms. The domestic firms are more like long-term contractors rather than active exporters, and their export decisions are less likely to be affected by uncertainty in foreign markets once the firms obtain contracts. Thus, in this paper I focus on transactions of ordinary trade, in which firms make their own decisions on production, pricing, and exporting, without being constricted by the existing contracts with foreign upstream suppliers. In particular, the model will be estimated using the data for the ceramics industry. This industry produces sanitation ceramics, special ceramics, and daily-used ceramics. This industry fits the study purpose well because these firms export a very concentrated product line. The major products for exporting are colorful dinnerware and ornamental articles of ceramics such as statuettes. This means that they can be viewed as single-product firms, each of which produces a differentiated product. More importantly, most export transactions in this industry are ordinary trade. In particular, in

2006, the ordinary trade in this industry generated revenue of more than 390 million US dollars, which accounts for 90% of all trade types. This suggests that uncertainty and learning about foreign demand are potentially important when firms make export decisions.

The second data source is the Chinese Annual Survey of Manufacturing from the Chinese National Bureau of Statistics. This data set provides detailed annual firm-level production information of all medium and large manufacturing firms that had total annual sales of more than $600,000 from 2000 to 2006. The primary variables include firm-level domestic revenue, labor wage, employment, material input, and capital stock. This information is used to construct a firm-time specific productivity measure, as well as a marginal production cost function with observable cost shifters such as capital stock and wage rate.

The sets of firm identification numbers are not the same in the two data sets since they are collected by different agencies. However, I am able to match the two data sets according to the recorded firm name, phone number, zip code, and some other identifying variables. About 114,000 out of 278,000 firms in the custom data set are matched (around 41%).[36] In the estimation of static parameters, I use the input and output information for all firms in the ceramics industry, but the estimation of dynamic parameters is based on the data for firms that can be identified from both data sets.

6 Empirical Results

I estimate the structural model developed above using shipment-level exports from the Chinese ceramics industry to Germany, supplemented with firm-level production data as described the last section. However, this estimation can be easily extended to other industries and foreign markets. I have chosen this particular industry because it fits the study purpose well for the aforementioned reasons. In the estimation, I use the 2000-2006 data on the 394 firms that can be identified from both the shipment-level export data set and the annual production data set.

[36]Note that the custom data records all transactions for all firms, while the annual survey data only records medium and large manufacturing firms, thus the percentage of matched firms conditional on medium and large manufacturing firms should be larger than 41%.

6.1 Estimates of Static Parameters and Productivity

The static parameters include the parameters in the marginal cost function (1), productivity evolution (2), and the demand function in foreign markets (8).

Table 2 shows the estimates of the marginal cost and productivity parameters. Note that coefficient γ_k measures the elasticity of capital in the marginal cost. The negative estimate (-0.063) means that when capital stock is larger, the marginal cost is lower. The estimate of γ_w is positive (0.056), which implies that firms with higher wage rates incur larger marginal cost in production. More importantly, the estimated parameters of the AR(1) productivity evolution are both positive and significant. The high estimate of g_1, 0.832, implies that productivity evolves persistently. Thus, firms with high productivity in this year will expect to have high productivity in the next year. However, the standard deviation of the innovation term is $\sigma_\epsilon = 0.095$. This implies that there are still significant unexpected productivity shocks that shift firm productivity. Thus, it is necessary to take the evolution of productivity into account in the investigation of the learning process.

Table 3 shows the estimates of the domestic demand elasticity as well as demand elasticities for the ten foreign markets that account for 61% of total exports. For simplicity, I assume the other markets share the same demand elasticity, which is represented by η^{oth}. This essentially simplifies the estimation equation to

$$TVC_{it} = (1 + \frac{1}{\eta^D})R_{it}^D + \sum_{j=1}^{10}(1 + \frac{1}{\eta^j})X_{it}^j + (1 + \frac{1}{\eta^{oth}})X_{it}^{oth} + \upsilon_{it}, \tag{50}$$

where υ_{it} is the measurement error. As shown in Table 3, this simplification does not pose any problem since the elasticities are very similar across the domestic market and all ten foreign markets.[37] These estimates indicate that the demand elasticities range from 3.96 to 4.18, implying markups of price over the marginal cost of 31.4 to 33.7 percent. Given that these markets spread all over the world, the result suggests that there is not much difference across different markets in terms of demand elasticity. An important implication is that firms are unlikely to face uncertainty about the demand elasticity in a foreign market. The reason is that the firm's operation in the domestic

[37] The result from Aw, Roberts, and Xu (2011) also shows nearly identical elasticity of the domestic market and the aggregated foreign market using Taiwanese electronics industry.

market or any foreign market is enough to allow them to know the demand elasticity across all other foreign markets. Thus, the assumption made throughout this paper – firms know the slope but have uncertainty about the intercept of the demand curve – is reasonable.

The estimated standard deviation of signal noise u_{it} for Germany is $\hat{\sigma}_u = 1.59$. It measures the the informativeness of each individual signal. If $\hat{\sigma}_u$ is high, then each transaction contains less effective information about the true demand factor ξ_i; accordingly, it takes the firm more transactions to reach a certain accuracy of the belief. In particular, the estimated $\hat{\sigma}_u = 1.59$ implies that the learning speed is slow and it takes 49 transactions to reduce the variance (σ_{it}^2) from 1 to 0.05.

6.2 Estimates of Dynamic Parameters

The dynamic parameters include the mean and standard deviation of the initial belief, the parameterized distributions of fixed and sunk costs, and parameters for the order process. The model is estimated with shipment-level export data from the Chinese ceramics industry to Germany from 2000 to 2006.

Since the data set only includes exports from year 2000, I do not observe firms' export status before 2000. Thus, I use year 2000 as an initial condition and allow firms to hold different beliefs when they first show up in the data, since some firms may have operated in the foreign market for a long time which I do not observe. In particular, I divide firms into two groups: potential entrants in the foreign market and experienced exporters, according to whether or not they exported to Germany in 2000.[38] The potential entrants are assumed to have an initial belief characterized by $N(m_{00}, \sigma_{00})$, while the experienced exporters are assumed to have an initial belief characterized by $N(m_{10}, \sigma_{10})$. These are initial beliefs that are held by firms when they first appear in the data set. The fixed and the sunk entry costs are assume to be drawn from Exponential distributions, with mean $\overline{c}^f$ for the fixed cost and mean $\overline{c}^s$ for the sunk cost, respectively. These distribution parameters are assumed to be the same for both potential entrants and experienced exporters.

[38] Here I assume firms within each group have the same initial belief. However, it is possible to allow initial belief to be heterogeneous within each group. For example, firm age, capital stock and ownership are observable in the data set and can be used to control for the heterogeneity in the initial belief by writing the initial belief as a function of these observable characteristics.

The estimate of (ψ_0, ψ_1, ψ_2) from the Maximum Likelihood estimation (47) is given in Table 4. All estimates are positive and significant. $\hat{\psi}_1 = 0.626$ implies that the persistence of the number of transactions is high. Firms with more transactions in the past period will also have more transactions in this period, if they choose to export. Moreover, the positive and significant estimate of ψ_2 suggests that firms that continue to export will expect around 65% more transactions in the current period than firms that did not export in the last period. This suggests that there is a significant "building-up" effect of customer stock that leads to export growth at firm level.

The estimates of initial beliefs and distribution parameters for the fixed and sunk costs are obtained from the Maximum Likelihood estimation (48) and are reported in Table 5. In particular, the estimates of initial beliefs for the two groups deliver two interesting implications. First, as expected, experienced exporters tend to have higher initial expectations, and this implies that their export participation is associated with high expectations about foreign demand. Second, experienced exporters face less uncertainty. This is reasonable since unlike potential entrants these firms may have operated in the foreign market for a long time and have learned a lot before year 2000. Notice that the estimate of standard deviation of signal noise $\hat{\sigma}_u = 1.59$ is significantly larger than the standard deviations of initial beliefs. This implies that the learning speed is slow. In particular, as shown in Figure 2, it takes 39 more transactions (signals) for a new exporter to reach the same accuracy (0.236) of the belief as an experienced exporter.

Export participation can be influenced by demand uncertainty through two different channels. The first channel is the variation of the expectation based on the observed signals (transaction outcomes). A high expectation implies a high expected value of exporting which encourages the firm to export. The second channel is the option value of learning. As shown in Figure 3, the value difference between the two choices, to export and not to export, is decreasing and concave as demand uncertainty disappears, holding other factors constant. As the firm exports for a longer time and conducts more transactions with foreign buyers, demand uncertainty resolves over time and the option value of learning from exporting decreases. Since the export decision relies on the comparison between this value difference and the fixed or sunk cost that the firm incurs, the decreasing option value of learning contributes to the stylized fact that many firms quit exporting within a short period after entry. However, this does not necessarily mean that all firms are less likely to export over time, as their updated expectations vary according to the signals received.

Table 6 compares the differences between exporters and non-exporters in terms of productivity and the demand expectation. Note that the expectation about the demand factor is $E(\exp(\xi)) = \exp(m+\sigma^2/2)$. I find that firms export if their productivity is high or if they have better expectation about foreign demand. In particular, the average productivity of exporters is 0.089, which means exporters are 16% (i.e. exp(0.145)) more efficient in production than non-exporters. Also, on average, $m + \sigma^2/2$ is higher for exporters (0.516) than non-exporters (-0.019). This implies that exporters expect 71% (i.e., exp(0.535)) more demand than non-exporters. It suggests that expected demand is indeed a determinant of the export decision.

However, the heterogeneity in productivity and the demand belief looks differently in potential entrants and experienced exporters. Figure 4 shows the kernel densities of productivity for potential entrants and experienced exporters respectively.[39] Although there is significant productivity heterogeneity within both groups, the mean difference between the two groups is small: 0.131. This implies that on average experienced exporters are 14% (i.e., exp(0.131)) more efficient in production than potential entrants. In turn, this suggests that experienced exporters expect 49% more profit resulted from high productivity, compared with potential entrants. Figure 5 shows the kernel densities of the demand belief (summarized as $m + \sigma^2/2$) for the two groups respectively. The result indicates substantial firm heterogeneity in the demand belief. Consistent with their initial beliefs, experienced exporters hold more optimistic and less dispersed beliefs about their demand, compared with potential entrants. More importantly, the demand belief difference between the two groups is large. In particular, on average experienced exporters expect 88% more profit resulted from high demand, compared with potential entrants. These comparisons suggest that although experienced exporters are superior to potential entrants in both the expected demand and productivity, the former is the dominant difference.

These findings suggest productivity and the demand belief play different roles in export participation for potential entrants and experienced exporters. In particular, a firm decides to export may because of high productivity or high demand expectation, both of which lead to high expected profit

[39] Productivity estimates across firms and over time are pooled together to obtain the densities.

of exporting. The estimated model allows me to decompose the expected profit difference between exporters and non-exporters into two components: a productivity component and a demand component. These two components can be further compared between potential entrants and experienced exporters. Table 7 shows the detailed comparison. To put productivity and the demand belief in the same scale of expected profit and thus render them comparable, I multiply productivity by $(\eta + 1)$ since productivity enters the expected profit as $(\eta+1)\omega$. It turns out that the average productivity difference (i.e., $\Delta(\eta + 1)\omega$) between exporters and non-exporters in the potential-entrant group is 0.259, and its counterpart in the experienced-exporter group is slightly larger (0.268). This suggests for both group, exporters expect about 30% more export profit than non-exporters because of higher productivity. On the other hand, the average expected demand difference between exporters and non-exporters in the potential-entrant group is 0.656 (corresponding to 93% more profit), but its counterpart for the experienced-exporter group is significantly smaller (0.226) (corresponding to 25% more profit). This implies that the major cross-sectional difference between exporters and non-exporters in the experienced-exporter group is productivity, but for the potential-entrant group, heterogeneity in expected demand is the dominant factor. This result is reasonable and expected, since experienced exporters face less uncertainty about their foreign demand thus productivity becomes the main determinant of export participation. However, potential entrants face much more demand uncertainty, which encourages them to enter into the market although their productivity is not as high as that of experienced exporters.

Above results indicate substantial heterogeneity in both productivity and the demand belief, and show that both processes are driving forces of export dynamics. However, the two processes play different roles for potential entrants and experienced exporters. In the next section, I employ the estimated model to investigate how the two forces affect export participation separately for both potential entrants and experienced exporters.

7 Counterfactual Analysis

I conduct two sets of counterfactual analysis about productivity evolution and the belief updating process to study how these processes influence the export participation of potential entrants and experienced exporters. In the first exercise, I shut down either evolution to evaluate how export participation is influenced by that process. In the second exercise, I experiment with the initial belief of potential entrants to study how it affects export participation.

I consider two scenarios in the first set of analysis. First, I shut down the belief evolution by assuming that each firm knows its demand factor and faces no uncertainty. The demand factors of potential entrants and experienced exporters are drawn from the distribution of each group, $N(\hat{m}_{00}, \hat{\sigma}_{00})$ and $N(\hat{m}_{10}, \hat{\sigma}_{10})$, respectively. Note that there is heterogeneity of the demand factor across firms, but the demand factor is fixed over time within a firm and there is no uncertainty. Moreover, I allow the productivity of each firm to evolve as assumed by the model. The predicted percentages of exporters in the two groups as well as all firms are shown in the third row of Table 8. The difference between the predicted percentage and actual percentage in data tells us how export participation is affected by heterogeneity in the demand belief. In particular, the percentages of exporters in potential entrants and experienced exporters decrease by 16% and 7%, respectively, and the percentage of exporters in all firms decreases by 12%. This means that the evolution of the demand belief is indeed a driving force of exporting.

The second scenario is to eliminate productivity evolution by replacing the productivity of each firm with its lowest productivity. Note that this only eliminates the time-series productivity heterogeneity within a firm, but it still allows for productivity heterogeneity across firms. I also leave the demand belief to evolve as assumed by the model. The predicted percentages of exporters are shown in the last row of Table 8. The difference between the predicted and actual percentages shows how export participation is affected by time-series productivity heterogeneity. The percentages of exporters in potential entrants and experienced exporters decrease by 3% and 6%, respectively, and the percentage for all firms decreases by 4%. This means that the evolution of productivity has a more significant effect on experienced exporters than potential entrants. However, the degrees by which the percentages decrease are less than those in the first scenario, where the demand belief evolution is controlled for. The comparison of the two scenarios implies that, for experienced exporters both forces significantly influence export participation while for potential entrants the belief evolution plays a more important role.

Of course, as shown in the second row of Table 8, the percentages of exporters predicted by the original model are very close to the actual percentages in data. This means the results in the above analysis are reliable.

In the second set of experiments, I adjust the initial belief of potential entrants to study how it affects export participation. Productivity still evolves as described in the model. In particular, I make the following three adjustments, and the results are summarized in Table 9. First, I assume that potential entrants have the same initial expectation as experienced exporters (which is higher), and leave their uncertainty unchanged. As a result, the percentage of exporters of potential entrants increases from 26% to 51%, almost doubled. This means the expectation of the belief does have a significant effect on export participation. Second, I adjust the standard deviation of the belief, which measures the demand uncertainty, to be the one of the experienced exporters (which is smaller), and let their expectations remain the same. As expected, the percentage of exporters decreases to 23% (which corresponds to a decrease of 11% in the number of exporters), since the option value of learning is smaller. Lastly, I make potential entrants have the same initial belief as experienced exporters. The model predicts that the percentage of exporters will increase to 40%. Note that although the increase in the expectation and the decrease in uncertainty have opposite effects, the combination of the two, in this case, has a net positive effect. That is, if potential entrants hold the same belief as experienced exporters, then they are more likely to export.

8 Conclusion and Discussion

In this paper, I develop a structural model of export dynamics with productivity evolution and Bayesian learning about demand in foreign markets to study how the two evolution processes affect firms' export participation. In particular, I allow firms to have inaccurate beliefs about foreign demand and to learn about it through their own individual export transactions. The model contributes to the current literature on export dynamics by allowing firms' export decisions to depend on two heterogenous factors, productivity and the demand belief: while productivity exogenously evolves over time, the demand belief is endogenously related to the export decision, and is updated periodically based on the demand signals received after exporting.

I then apply the model to the shipment-level export data set of the Chinese ceramics industry to investigate the magnitude of uncertainty and examine how the resolution of uncertainty and productivity evolution affect export participation. A general finding is that exporters and non-exporters are different in both productivity and the demand belief. The demand belief heterogeneity is the dominant difference between potential entrants in export markets and experienced exporters. Moreover, productivity and the demand belief play different roles for potential entrants and experienced exporters. The major cross-sectional difference between exporters and non-exporters for experienced firms is productivity, but for potential entrants, demand uncertainty is the dominant factor.

In the counterfactual analysis, I investigate how the two evolution processes influence export dynamics. The result confirms that, the learning process is more important for potential entrants while for experienced exporters both productivity and the learning process are driving forces of export participation. In particular, it predicts that reducing the level of uncertainty of potential entrants to that of experienced exporters causes the number of exporters to fall by 11%. It also show that the learning mechanism indeed contributes to the large attrition rate after the first year of exporting and gradually stabilized export decisions observed in data.

This paper demonstrates that both productivity evolution and the learning process contribute to the observed export dynamics, and shows how to utilize shipment-level exports and firm-level production information to estimate the role of each process. These empirical results are obtained from studying the exports of the Chinese ceramics industry to Germany. However, much about what I claim for this industry can be generalize to other destinations and industries. With the results for more destinations and industries, it is possible to connect export dynamics and the learning process to destination-specific and industrial-specific characteristics.

This paper also implies promising future research. One direction is to examine how the productivity improvement is endogenously related to Bayesian learning about foreign demand. That is, instead of assuming that productivity exogenously evolves over time, the extension recognizes that firms may improve their productivity levels by conducting R&D investment. On the one hand, firms export decisions are driven by both their demand beliefs and productivity; on the other hand, firms with high demand expectations may choose to conduct R&D which in turn increases the propensity of exporting and thus has a further impact on their demand beliefs. In this circumstance, there could be a positive linkage between the demand belief and R&D investment. As the first step, I will examine this linkage using reduced form regressions with data on firm-level production, R&D, and shipment-level exports. Then I will take the linkage into a structural dynamic model to further examine how a forward-looking firm's choice of R&D is related to its belief about foreign demand.

Another interesting extension would be to study additional dynamic patterns of exporting firms such as the sequential exporting documented in Albornoz, Calvo Pardo, Corcos, and Ornelas (2012): firms often start selling to a single country; if it is successful, they tend to expand to other markets. My model can capture this feature by allowing that learning processes across different markets are correlated. As a result, the export outcomes observed in one market are informative regarding the demand in another market. The potential contribution of this extension to the current literature is two-fold. First, it is able to rationalize the gradually stabilized export dynamics through Bayesian learning. Second, it takes productivity evolution into account in the analysis. This is important since without controlling for productivity evolution, it is likely to attribute the effect from the productivity shock to the role of Bayesian learning. However, the major difficulty of this extension is computational: there are more belief state variables to keep track of in the estimation. One possible simplification is to assume the firm's demand is at global scope rather than market-specific. This is left as future work.

References

Ackerberg, D. A. (2003): "Advertising, learning, and consumer choice in experience good markets: an empirical examination," *International Economic Review*, 44(3), 1007–1040.

Albornoz, F., H. F. Calvo Pardo, G. Corcos, and E. Ornelas (2012): "Sequential exporting," *Journal of International Economics*, 88(1), 17–31.

Arkolakis, C. (2010): "Market Penetration Costs and the New Consumers Margin in International Trade," *Journal of political economy*, 118(6), 1151–1199.

Aw, B. Y., M. Roberts, and D. Y. Xu (2011): "R&D Investment, Exporting, and Productivity Dynamics," *American Economic Review*, 101, 1312–1344.

Baily, M. N., C. Hulten, D. Campbell, T. Bresnahan, and R. E. Caves (1992): "Productivity dynamics in manufacturing plants," *Brookings papers on economic activity. Microeconomics*, 1992, 187–267.

Baldwin, J. R., and P. Gorecki (1998): *The dynamics of industrial competition: A North American perspective*. Cambridge University Press.

Bartelsman, E., and M. Doms (2000): "Understanding Producivity: Lessons from Longitudinal Microdata," *Journal of Economic Literature*, 38(3), 569–594.

Bastos, P., and J. Silva (2010): "The quality of a firm's exports: Where you export to matters," *Journal of International Economics*, 82(2), 99–111.

Bernard, A. B., J. Eaton, J. B. Jensen, and S. Kortum (2003): "Plants and Productivity in International Trade," *American Economic Review*, pp. 1268–1290.

Crawford, G. S., and M. Shum (2005): "Uncertainty and learning in pharmaceutical demand," *Econometrica*, 73(4), 1137–1173.

Das, S., M. J. Roberts, and J. R. Tybout (2007): "Market Entry Costs, Producer Heterogeneity and Export Dynamics," *Econometrica*, 75(3), 837–873.

DeGroot, M. H. (2005): *Optimal statistical decisions*, vol. 82. Wiley-Interscience.

Doraszelski, U., and J. Jaumandreu (2013): "R&D and productivity: Estimating endogenous productivity," *The Review of Economic Studies*.

Eaton, J., M. Eslava, D. Jinkins, C. J. Krizan, and J. Tybout (2013): "A Search and Learning Model of Export Dynamics," Working Paper, Department of Economics, The Pennsylvania State University.

Eaton, J., S. Kortum, and F. Kramarz (2011): "An anatomy of international trade: Evidence from French firms," *Econometrica*, 79(5), 1453–1498.

Foster, L., J. Haltiwanger, and C. Syverson (2008): "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability?," *American Economic Review*, 98(1), 394–425.

Freund, C., and M. Pierola (2010): "Export entrepreneurs: evidence from Peru," *World Bank Policy Research Working Paper Series, Vol.*

Helpman, E., M. J. Melitz, and S. R. Yeaple (2004): "Export versus FDI with Heterogeneous Firms," *The American Economic Review*, 94(1), 300–316.

Irarrazabal, A. A., and L. D. Opromolla (2006): "Hysteresis in export markets," *New York University, Working Paper.*

Levinsohn, J., and A. Petrin (2003): "Estimating Production Functions Using Inputs to Control for Unobservables," *The Review of Economic Studies*, 70(2), pp. 317–341.

Melitz, M. J. (2003): "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 71(6), 1695–1725.

Nagypál, É. (2007): "Learning by doing vs. learning about match quality: Can we tell them apart?," *The Review of Economic Studies*, 74(2), 537–566.

Nguyen, D. X. (2012): "Demand uncertainty: Exporting delays and exporting failures," *Journal of International Economics*, 86(2), 336–344.

Olley, G. S., and A. Pakes (1996): "The Dynamics of Productivity in the Telecommunications Equipment Industry," *Econometrica*, 64(6), pp. 1263–1297.

Rauch, J. E., and J. Watson (2003): "Starting small in an unfamiliar environment," *International Journal of Industrial Organization*, 21(7), 1021–1042.

Roberts, M., and J. Tybout (1997): "The Decision to Export in Colombia: An Empirical Model of Entry with Sunk Costs," *American Economic Review*, 87, 545–564.

Roberts, M., D. Xu, X. Fan, and S. Zhang (2012): "A Structural Model of Demand, Cost, and Export Market Selection for Chinese Footwear Producers," NBER Working Paper 17725.

Rust, J. (1987): "Optimal Replacement of GMC Bus Engines: An Empirical Model of Harold Zurcher," *Econometrica*, 55(5), 999–1033.

Stevenson, R. (1980): "Measuring technological bias," *The American Economic Review*, 70(1), 162–173.

Syverson, C. (2011): "What Determines Productivity?," *Journal of Economic Literature*, 49(2), 326–65.

Appendices

Appendix A Solve for Value Function

I follow the method implemented in Nagypál (2007) to compute the value function. The first step in implementing the method is to solve for the value function at the limit state ($\sigma_{it} = 0$), i.e., the value function when the firm knows its exact ξ_i. This can be achieved by value function iteration. Specifically, I omit the notation for firm i and denote the new state variables without σ_{it} as $\tilde{s}_t = (\tilde{\phi}, \tilde{n}_{t-1}, \omega_t, m)$. Note that at the limit state, the firm knows its own demand factor, so there is no transition of state m. Now the value function $V(\tilde{s}_t, e_{t-1})$ with no uncertainty can be computed by iterating the following Bellman equations implied by equation 21:

$$\begin{aligned}V(\tilde{s}_t, 0) = \Pr &\Big[E\Pi(\tilde{s}_t, 0) - c^s + \delta EV(\tilde{s}_{t+1}, 1) < \delta EV(\tilde{s}_{t+1}, 0)\Big] \times \delta EV(\tilde{s}_{t+1}, 0) \\ &+ \Pr \Big[E\Pi(\tilde{s}_t, 0) - c^s + \delta EV(\tilde{s}_{t+1}, 1) > \delta EV(\tilde{s}_{t+1}, 0)\Big] \\ &\times \Big[E\Pi(\tilde{s}_t, 0) - E(c^s|e_t = 1) + \delta EV(\tilde{s}_{t+1}, 1)\Big]\end{aligned} \tag{51}$$

and

$$\begin{aligned}V(\tilde{s}_t, 1) = \Pr &\Big[E\Pi(\tilde{s}_t, 1) - c^f + \delta EV(\tilde{s}_{t+1}, 1) < \delta EV(\tilde{s}_{t+1}, 0)\Big] \times \delta EV(\tilde{s}_{t+1}, 0) \\ &+ \Pr \Big[E\Pi(\tilde{s}_t, 1) - c^f + \delta EV(\tilde{s}_{t+1}, 1) > \delta EV(\tilde{s}_{t+1}, 0)\Big] \\ &\times \Big[E\Pi(\tilde{s}_t, 1) - E(c^f|e_t = 1) + \delta EV(\tilde{s}_{t+1}, 1)\Big].\end{aligned} \tag{52}$$

where $E\Pi(\tilde{s}_t, 1)$ is the period export profit implied by equation 20, and

$$\begin{aligned}&EV(\tilde{s}_{t+1}, e_t) \\ &\equiv E[V(\tilde{s}_{t+1}, e_t)|\tilde{s}_t, e_{t-1}] \\ &= \sum_n \left[\Pr(\tilde{n}_t = n|s_{it}, e_{it-1}) \int V(\tilde{s}_{t+1}, e_t) dF_\omega(\omega_{t+1}|\omega_t) \right],\end{aligned} \tag{53}$$

where $F_\omega(\cdot)$ and $\Pr(\cdot|s_{it}, e_{it-1})$ are the transition probabilities of ω_t and $\tilde{n}_t$, respectively.

Then $V(\tilde{s}_t, e_{t-1})$ is used as an approximation of the value function with low enough uncertainty (i.e., a small enough σ_{it}). Note that the standard deviation σ_{it} is a decreasing function of the number of signals, and there is a one-to-one mapping between the σ_{it} and the number of signals, given the initial state of σ_{it} (i.e., σ_{i0} in the initial belief). Thus, we use the number of signals as an index of uncertainty. I denote $V(\tilde{s}_t, e_{t-1}, N)$ as the value function after receiving a total of N signals. Denote N^* such that $\sigma_{iN^*} < \epsilon$ as the cutoff number of signals after which the value function with uncertainty is approximated by the value function with no uncertainty. That is, for $N \geq N^*$,

$$V(\tilde{s}_t, e_{t-1}, N) \approx V(\tilde{s}_t, e_{t-1}). \tag{54}$$

Now I have the value function at one level of uncertainty: $V(\tilde{s}_t, e_{t-1}, N^*) = V(\tilde{s}_t, e_{t-1})$. Then I can recover the value function with an arbitrary higher uncertainty ($V(\tilde{s}_t, e_{t-1}, N)$ with $N \leq N^*$)

using backward induction, which is implied from the Bellman equation:

$$
\begin{aligned}
V(\tilde{s}_t, 0, N_t) = \Pr\Big[&E\Pi(\tilde{s}_t, 0, N_t) - c^s + \delta EV(\tilde{s}_{t+1}, 1, N_{t+1}|N_t) < \delta EV(\tilde{s}_{t+1}, 0, N_{t+1}|N_t)\Big] \\
&\times \delta EV(\tilde{s}_{t+1}, 0, N_{t+1}|N_t) \\
&+ \Pr\Big[E\Pi(\tilde{s}_t, 0, N_t) - c^s + \delta EV(\tilde{s}_{t+1}, 1, N_{t+1}|N_t) > \delta EV(\tilde{s}_{t+1}, 0, N_{t+1}|N_t)\Big] \\
&\times \Big[E\Pi(\tilde{s}_t, 0, N_t) - E(c^s|e_t = 1) + \delta EV(\tilde{s}_{t+1}, 1, N_{t+1}|N_t)\Big],
\end{aligned} \tag{55}
$$

and

$$
\begin{aligned}
V(\tilde{s}_t, 1, N_t) = \Pr\Big[&E\Pi(\tilde{s}_t, 1, N_t) - c^f + \delta EV(\tilde{s}_{t+1}, 1, N_{t+1}|N_t) < \delta EV(\tilde{s}_{t+1}, 0, N_{t+1}|N_t)\Big] \\
&\times \delta EV(\tilde{s}_{t+1}, 0, N_{t+1}|N_t) \\
&+ \Pr\Big[E\Pi(\tilde{s}_t, 1, N_t) - c^f + \delta EV(\tilde{s}_{t+1}, 1, N_{t+1}|N_t) > \delta EV(\tilde{s}_{t+1}, 0, N_{t+1}|N_t)\Big] \\
&\times \Big[E\Pi(\tilde{s}_t, 1, N_t) - E(c^f|e_t = 1) + \delta EV(\tilde{s}_{t+1}, 1, N_{t+1}|N_t)\Big].
\end{aligned} \tag{56}
$$

where N_t and N_{t+1} are the total number of signals received up to the beginning of period t and $t+1$, and $EV(\tilde{s}_{t+1}, 0, N_{t+1}|N_t)$ and $EV(\tilde{s}_{t+1}, 1, N_{t+1}|N_t)$ are short-hand notations for the expected value functions, which are explained as follows.

If the firm decides not to export in period t, then no additional signal will be received. That is, $N_{t+1} = N_t$, and there is no transition of m_{t+1} or $\tilde{n}_t$: $m_{t+1} = m_t$ and $\tilde{n}_t = \tilde{n}_{t-1}$. Thus,

$$
\begin{aligned}
&EV(\tilde{s}_{t+1}, 0, N_{t+1}|N_t) \\
&= \int V(\tilde{s}_{t+1}, 0, N_t) dF_\omega(\omega_{t+1}|\omega_t).
\end{aligned} \tag{57}
$$

But if the firm decides to export in period t, then it may receive a total of n_t additional signals in period t. The total number of signals will increase from N_t to $N_{t+1} = N_t + n_t$. Note that n_t becomes the most recent number of transactions: $\tilde{n}_t = n_t$. The transition of the belief state specified in equation 31 implies:

$$
\begin{aligned}
&EV(\tilde{s}_{t+1}, 1, N_{t+1}|N_t) \\
&= E[V(\tilde{s}_{t+1}, 1, N_{t+1})|\tilde{s}_t, e_{t-1}, N_t] \\
&= \sum_{n_t=1}^{\infty} \Pr(n_t|\tilde{s}_t, e_{t-1}) E[V(\tilde{s}_{t+1}, 1, N_t + n_t)|\tilde{s}_t, e_{t-1}, N_t] \\
&= \sum_{n_t=1}^{\infty} \left[\Pr(n_t|\tilde{s}_t, e_{t-1}) \int V(\tilde{s}_{t+1}, 1, N_t + n_t) dF_\omega(\omega_{t+1}|\omega_t) dF_m(m_t|n_t, m_{t-1}, N_t, e_t = 1)\right],
\end{aligned} \tag{58}
$$

where $\Pr(n_t|\tilde{s}_t, e_{t-1})$, $F_\omega(\omega_{t+1}|\omega_t)$ and $F_m(m_t|n_t, m_{t-1}, N_t, e_t = 1)$ are the transition probabilities of the number of transactions, the productivity and the expectation in the belief state respectively, each of which is given by equation 30, 23, and 28.

In this way, I recover all the relevant value function with uncertainty, which is indexed by N.

Figure 1: Percentage of export status switches

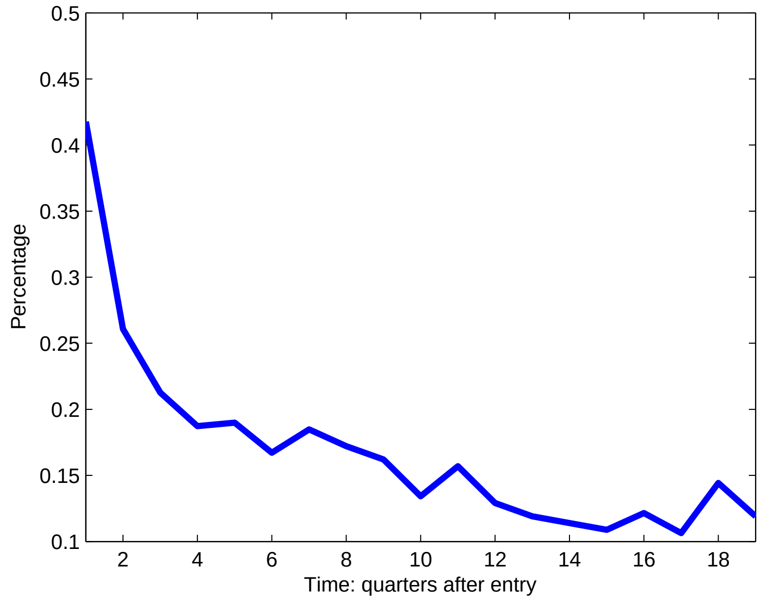

Figure 2: The resolution of uncertainty of demand

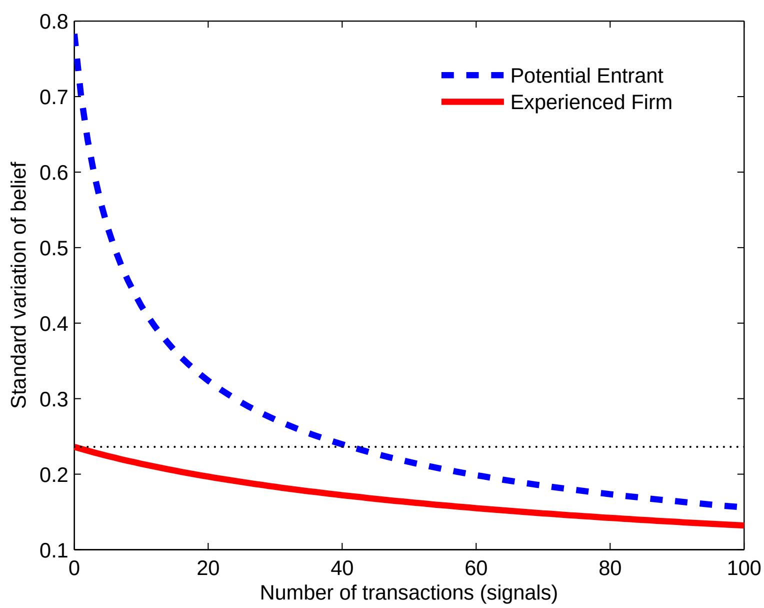

Figure 3: Value difference between export and not to export

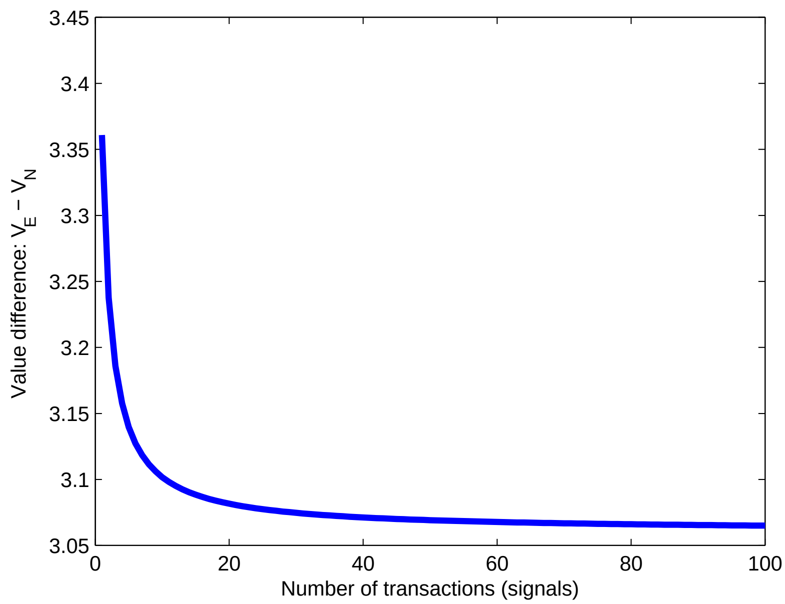

Figure 4: Heterogeneity in Productivity, by Group

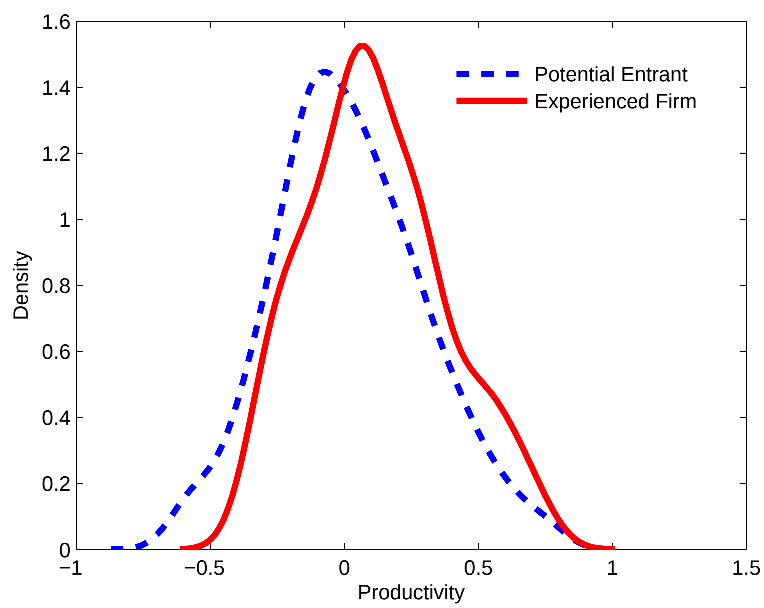

Figure 5: Heterogeneity in Expected Foreign Demand, by Group

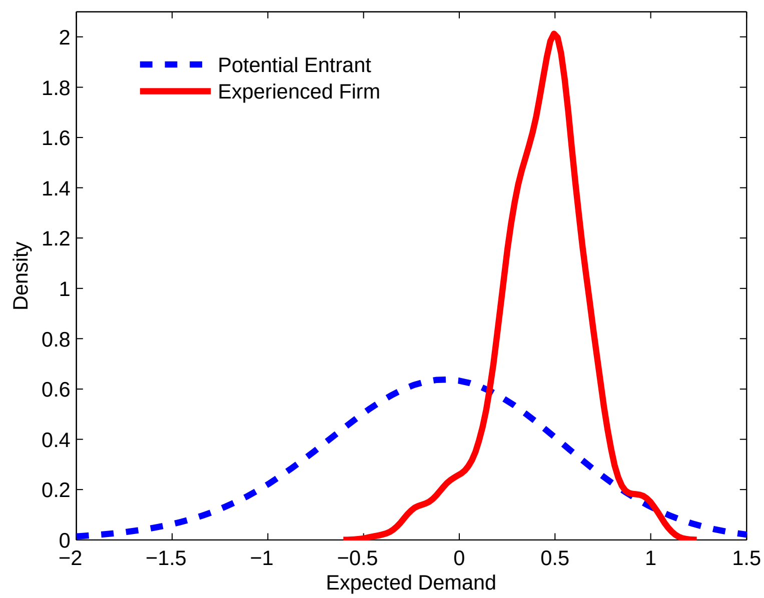

Table 1: Percentage of exporters to U.S., by entry cohort

	Year of entry			
Year	2001	2002	2003	2004
2001	100%	-	-	-
2002	76%	100%	-	-
2003	74%	70%	100%	-
2004	75%	68%	65%	100%
2005	76%	72%	65%	68%
2006	75%	68%	60%	69%

[1] Percentages are based on the total number of firms in each cohort. For example, 75% in the second column and sixth row means that among firms entered in year 2001, 75% of them exported in year 2004.

[2] All transaction types are included.

Table 2: Estimates of marginal cost and productivity parameters

Parameter	Estimate	Std. Err.
γ_w	0.056^{***}	(0.003)
γ_k	-0.063^{***}	(0.003)
g_0	0.055^{***}	(0.010)
g_1	0.832^{***}	(0.005)
R^2	0.87	
#Obs	6613	

[1] The parameters are estimated for Ceramic Industry from Chinese Annual Survey of Manufacturing. Each observation is a year.

Table 3: Demand elasticity estiamtes

Parameter	Estimate	Std. Err.
$1+1/\eta^{D}$	0.758^{***}	(0.005)
$1+1/\eta^{USA}$	0.753^{***}	(0.005)
$1+1/\eta^{ITA}$	0.753^{***}	(0.005)
$1+1/\eta^{JPN}$	0.749^{***}	(0.005)
$1+1/\eta^{DEU}$	0.753^{***}	(0.006)
$1+1/\eta^{ARE}$	0.756^{***}	(0.005)
$1+1/\eta^{GBR}$	0.752^{***}	(0.005)
$1+1/\eta^{ESP}$	0.748^{***}	(0.005)
$1+1/\eta^{AUS}$	0.760^{***}	(0.006)
$1+1/\eta^{CAN}$	0.761^{***}	(0.006)
$1+1/\eta^{HKG}$	0.756^{***}	(0.005)
$1+1/\eta^{oth}$	0.755^{***}	(0.005)
R^2	0.97	
#Obs	2947	

[1] The parameters are estimated for Ceramic Industry for matched firms both in Chinese Annual Survey of Manufacturing and Chinese Customs Transactions. Each observation is a year.

Table 4: Estimates of Dynamic parameters (1)

Parameter	$\hat{\psi}_0$	$\hat{\psi}_1$	$\hat{\psi}_2$
Estimate	0.341^{***}	0.626^{***}	0.502^{***}
Std. Err.	(0.009)	(0.028)	(0.100)

Table 5: Estimates of Dynamic parameters (2)

Parameter	$\hat{m}_{00}$	$\hat{m}_{10}$	$\hat{\sigma}_{00}$	$\hat{\sigma}_{10}$	$\hat{\bar{c}}^{f}$	$\hat{\bar{c}}^{s}$
Estimate	-0.344	0.516	0.783	0.236	0.654	3.155
Std. Err.	(0.083)	(0.140)	(0.107)	(0.051)	(0.072)	(0.308)

Table 6: Export decision: productivity v.s. expected demand

	ω	$m + \sigma^2/2$
Export	0.089	0.516
Not to Export	-0.056	-0.019
Difference	0.145	0.535

[1] Numbers in the table are average values. Each observation is a firm-year combination.

Table 7: Within group comparison: profitability decomposition

Difference	$\Delta(\eta + 1)\omega$	$\Delta(m + \sigma^2/2)$
Potential Entrants	0.259	0.656
Experienced Exporters	0.268	0.226

[1] Numbers in the table are average differences between exporters and non-exporters. Each observation is a firm-year combination.

Table 8: Counterfactuals: learning v.s. productivity

Restriction	Pr(**export**|**potential**)	Pr(**export**|**experienced**)	Pr(**export**)
Data	26%	75%	39%
Model	26%	74%	39%
No Uncertainty	22% (↓ 16%)	69% (↓ 7%)	34% (↓ 12%)
No Prod. Evolution	25% (↓ 3%)	70% (↓ 6%)	38% (↓ 4%)

Table 9: Counterfactuals: role of initial belief

	Data	Model	Adjusted Expectation	Adjusted Uncertainty	Both
Pr(export)	26%	26%	51% (↑ 96%)	23% (↓ 11%)	40% (↑ 54%)