A Structural Model of Productivity, Uncertain Demand, and Export Dynamics*

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Abstract

Existing international trade literature focusing on either productivity or demand learning individually has documented them as driving forces of firm export decisions. In this paper, I incorporate both of them into a dynamic structural model of export to empirically quantify how the two mechanisms determine the firm-market level export participation separately. In this model, firms face uncertainty about their own foreign demand and gradually update their beliefs based on individual export transactions. I utilize an integrated data set on firm-level production and transaction-level exports of Chinese ceramics and glass industry to Germany to estimate the model. The empirical results indicate substantial firm heterogeneity in both productivity and the demand belief. Exporting firms have both higher productivity and demand expectation compared with non-exporting firms, with the difference of demand expectation being larger. For experienced exporters, productivity evolution is the major driving force of export participation; while for potential entrants in the foreign market, the learning process plays a more important role than that of experienced exporters. A counterfactual exercise implies that the trade cost reduction after China's accession of WTO further boosts export via providing the firms with the opportunity of learning about demand in foreign markets.

Keywords: *demand uncertainty, Bayesian learning, productivity, export dynamics*

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1 Introduction

International trade literature focusing on either productivity or demand learning individually has documented them as driving forces of firm export decisions. For example, Melitz (2003), Bernard, Eaton, Jensen, and Kortum (2003), Eaton, Kortum, and Kramarz (2011), Aw, Roberts, and Xu (2011) have studied how firm-level exports are driven by heterogeneous firm productivity. These papers predict firms with higher productivity are self-selected to exporting markets. Inspired by dynamic patterns such as the high attrition rate after entry, a growing strand of trade models have emphasized the roles of demand uncertainty and learning in the exporting markets: firms update beliefs regarding their own demand after exporting and they take this into account when making export decisions (Rauch and Watson, 2003; Freund and Pierola, 2010; Albornoz, Calvo Pardo, Corcos, and Ornelas, 2012; Nguyen, 2012; Eaton, Eslava, Jinkins, Krizan, and Tybout, 2013; Arkolakis, Papageorgiou, and Timoshenko, 2015; Timoshenko, 2015; Berman, Rebeyrol, and Vicard, 2015). However, without putting the demand learning process and time-varying productivity in the same framework, these papers are not able to empirically isolate their individual roles in the export decisions. For example, without controlling for the dynamics of firm productivity, it is likely to attribute the self-selected export resulted from an increase of productivity to the role of learning about high demand. Thus, identifying the roles of the two forces and empirically quantifying their mechanisms in export dynamics remain as empirical questions.

In this paper, I bridge the two strands of literature by estimating a structural model of export dynamics to quantitatively disentangle how firms' export decisions separately depend on productivity evolution and Bayesian learning about demand in foreign markets. First, I develop a single-agent infinite-horizon dynamic model of export decisions in which the firm faces uncertainty about its own foreign demand and gradually learns about it from export transactions according to Bayes' rule (in the spirit of Jovanovic, 1982). In particular, its demand belief evolves endogenously as the firm exports: the size of each individual export transaction influences the firm's expectation about its own foreign demand; the number of transactions determines how fast the demand uncertainty resolves. Second, I take the other driving force, productivity evolution, into account when I assess the role of the learning process. That is, the firm's export decision depends on both productivity and its demand belief. In order to empirically quantify their relative importance, I estimate the model with an integrated data set from Chinese ceramics and glass industry, by recovering productivity from firm-level production information and the learning process from shipment-level exports. This study is of particular interest in the context of Chinese export: new opportunities of exporting, rising together with great demand uncertainty in foreign markets, opened up for potential entrants when China joined WTO in 2001; at the same time, this came with significant productivity growth of Chinese manufacturing firms. Drawing data from 2000 to 2006, this paper sheds light on how productivity and demand learning contribute to the boom of Chinese export separately.

The recent literature has documented firm-level demand heterogeneity as a determinant of firm behavior (e.g., Foster, Haltiwanger, and Syverson, 2008; Pozzi and Schivardi, 2012). In the export context, Roberts, Xu, Fan, and Zhang (2012) find substantial heterogeneity in both demand and cost dimensions with demand being more dispersed. My paper builds on this insight: firms face heterogeneous demand in foreign markets. But the difference is that they have uncertainty about foreign demand, thus it is the belief about its own demand, rather than the demand itself, that directly influences a firm's export decision. Due to this uncertainty, a forward-looking firm update its demand belief after exporting. In this way, the export decision and belief updating are endogenously correlated: the firm makes the export decision based on its current demand belief, and it knows that current export experience updates the belief which affects its future export decisions.

However, the assessment of the role of the learning process is potentially biased if the evolution of productivity is ignored. Productivity has been recognized as distinct heterogeneity in studies of firm performance (e.g., Hopenhayn, 1992; Baily, Hulten, and Campbell, 1992; Baldwin and Gorecki, 1998; Irarrazabal and Opromolla, 2006; Luttmer, 2007),[2] as well as in explaining firm-level export (e.g., Helpman, Melitz, and Yeaple, 2004; Arkolakis, 2011). In particular, Eaton, Kortum, and Kramarz (2011) show that over half of the variation across firms in market entry can be attributed to productivity heterogeneity. From the dynamic perspective, Aw, Roberts, and Xu (2011) find that R&D and exporting have a positive effect on the firm's future productivity, which in turn drives more firms to self-select into both activities. Building upon the documented importance of productivity evolution, I allow firms' export decisions to depend on two types of evolving firm heterogeneity: productivity and the demand belief. The key is to identify the role of the learning process from the effect of productivity evolution, since neither of them is observable to researchers.

[2]See Bartelsman and Doms (2000) and Syverson (2011) for reviews.

Nonetheless, this paper faces the same challenge of the lack of output price data as in the literature, so the revenue productivity inverted from domestic sales captures both production efficiency and heterogeneous domestic demand. To address this issue, I model the demand learning as a process of learning about the relative difference between the firm's domestic demand and foreign demand. Given that most firms have operated in the domestic market for years,[3] it is plausible to assume that firms know their domestic demand. In turn, in the firm's point of view, the learning of the level of foreign demand is equivalent to the learning of the demand difference. This transformation enables me to use revenue productivity directly in modelling export decisions without recovering the physical production efficiency which requires either the output price data or a no-domestic-demand-heterogeneity assumption. The idea is that revenue productivity influences the export decision by giving the firm a reference foreign profitability based on its performance in the domestic market; the belief of the demand difference affects the export decision by adjusting the reference towards the true foreign profitability based on export experience. Specifically, I model the demand heterogeneity as constant demand factors, with the difference that the domestic demand is observed by the firm while the foreign demand is unknown. Conditional on exporting, the revenue in each individual transaction relative to the domestic sale reflects a signal of the *difference* of demand factors. The signal is noisy because the firm cannot tell whether a big export sale is due to a persistent high demand or simply being lucky. But the firm can update its belief based on the signals following Bayes' rule, which guides the firm to make its export decision in the next period.

Consequently, the identification and estimation strategy uses the above observation: domestic revenue is only affected by productivity while export participation is influenced by both productivity and the demand belief. As another novelty, I draw data from two sources to empirically disentangle the individual role of each process. The Chinese Annual Survey of Manufacturing provides firm-level production information while the Chinese Customs Transactions contains every individual shipment-level exports. I utilize data from the domestic market to recover the time-varying productivity for each firm. Meanwhile, shipment-level exports contain information on how Bayesian learning occurs, and allow me to construct demand signals and use them to update firms' demand beliefs in each period. A model with only productivity heterogeneity predicts more productive

[3] In the data used for estimation, the median firm age is seven years in 2001 (as a measure of domestic experience); in contrast, the opportunity of exporting opened up for most firms after China's accession to WTO in 2001, which implies zero experience for most of them up to 2001.

firms export. While in my model with the two-dimensional heterogeneity, firms that face greater uncertainty have higher demand expectation thus may also export, even if their productivity is not high. In turn, with both productivity and demand beliefs recovered, the cross-sectional and time series variations of export decisions identify the role of each driving force. Specifically, I estimate a discrete choice model of export participation via the Maximum Likelihood Method to recover trade cost distributions and prior beliefs, after firm-level productivity, its evolution process, and demand signals are estimated. To address the challenge of computational burden in this high-dimention-state dynamic model, I follow the recent but pioneer practice in Barwick and Pathak (2015) to use Sieve functional approximation combined with MPEC developed in Su and Judd (2012) in order to evaluate the value function of this dynamic model without solving it.

In the estimation, I utilize the data of the Chinese ceramics and glass industry integrated from the two sources during 2000 to 2006. Most export transactions in this industry are ordinary trade, in which firms make their own decisions of production, pricing, and exporting, without facing constraints from existing contracts with foreign companies (as opposed to processing trade). As a result, demand uncertainty, which may come from how foreign customers perceive the product appeal, is an important issue for the firms to consider at their export decisions. I focus on the exports to Germany to isolate the roles of productivity and the learning process at the firm-market level. To control for the initial condition, I divide firms into two groups, potential entrants to the German market and experienced exporters in the German market. The empirical results indicate substantial firm heterogeneity in both productivity and the demand belief. The dominant difference between exporting and non-exporting firms lies in the demand belief. Considering productivity is measured from the domestic sales, this suggests that although the domestic performance is positively correlated to export participation, it is the foreign demand that serves as the main predictor of the success in the German market. In cross-section comparison with potential entrants, experienced exporters have higher productivity and demand expectations, more transactions, and face less uncertainty, with the difference of demand expectation being the largest.

From the dynamic perspective, I use the estimated model to conduct counterfactual exercises and quantify the individual roles of the two drivers in determining export participation over time. In a set of restrictive counterfactuals, I shut down either the evolution of productivity or belief updating in order to evaluate how export participation is influenced by each process. The comparison of the results from the two scenarios with those generated from the full model shows that, for experienced exporters, productivity evolution is the major driving force of export participation, while for potential entrants, the learning process plays a more important role than that of experienced exporters. Also, the significant drop of export participation after all the learning elements are eliminated suggests evidence of the general existence of demand learning. This complements Dickstein and Morales (2015), which measure the information set of firms at the time of their export decisions. Moreover, the comparison of the initial and updated beliefs suggests entrants who received favorable demand signals continued exporting and survived in the German market. This further implies an indirect effect of the trade cost reduction via demand learning: when the trade costs are reduced by 10% permanently, I find the learning process drives the export probability of potential entrants from 13.5%, which a non-learning model would predict, further to 16.2%. The implication is that the observed export boom of Chinese firms after joining WTO may be also attributed to the opportunity of learning about demand in foreign markets in addition to the direct effect of trade cost reduction. This also collaborates with Arkolakis, Papageorgiou, and Timoshenko (2015) which develop a general equilibrium model of firm growth with learning about foreign demand and show that fixed cost subsidies to young firms can prevent early exit and thus enhance welfare.

This paper complements a vibrant literature in firm exporting and Bayesian learning of demand.[4] In particular, Timoshenko (2015) explains export product switching in a model of learning about appeal indices. Berman, Rebeyrol, and Vicard (2015) document demand learning as an important driver of post-entry export decisions by using detailed French exporter-level data. Relative to these papers, I contribute by quantitatively evaluating the relative importance of demand learning and productivity evolution in determining firm-market export in the dynamic dimension with the integrated Chinese data of both firm-level production and transaction-level exports.

This paper is organized as follows. Section 2 develops a structural model of export dynamics with heterogeneity in both productivity and Bayesian learning. Section 3 outlines an identification and estimation strategy. Section 4 describes the data for the estimation. The results are discussed in Section 5. Section 6 conducts counterfactual exercises, and I conclude in Section 7.

[4] This paper also fits into a broader empirical literature in the study of uncertainty and learning in a structural model. For example, see Ackerberg (2003) for a dynamic learning model to study both "informative" and "prestige" effects of advertising, Abbring and Campbell (2003) for a structural model of firm growth, learning, and survival for bars in Texas, and Crawford and Shum (2005) for a study of effects of uncertainty and learning in the demand for pharmaceutical drugs.

2 The Model

In this section, I develop a structural model of firm-market-level export dynamics with heterogeneous firms learning about their own demand in foreign markets. The feature of the model is that it considers how the two processes, productivity evolution and demand belief updating, affect a firm's export participation. I first provide an overview of the model then specify the details.

2.1 An Overview

Consider an industry with I single-product firms and an infinite-period horizon. There are a total of J foreign markets in addition to the domestic market. Firms are heterogeneous in both cost and demand dimensions. In the cost dimension, firms are different in productivity, a measure of efficiency in converting input to output. Higher productivity implies lower marginal cost in production. In the demand dimension, the locations of the demand curves are heterogeneous across firms, due to the difference in customer tastes and the product quality perceived by consumers in different markets. I refer to this heterogeneity as the firm's demand factor. The demand factor is allowed to be different across markets for each firm, since consumer tastes and competitors may be different across markets. For each firm, productivity evolves exogenously from one period to another,[5] but the demand factor in each market is time-invariant.

In each period t, each firm i makes decisions of production, pricing, and export participation in each foreign market, in addition to the domestic market.[6] Two types of information are crucial for it to make these decisions. The first one is its productivity level, and the second is its demand factor in each foreign market. At the beginning of each period t, the firm observes the realization of its productivity ω_{it} and the domestic demand. However, in contrast to the current literature (e.g., Roberts, Xu, Fan, and Zhang, 2012), this paper assumes the foreign demand factor is firm-market

[5]If firms endogenously choose productivity, by investing in R&D for example, then more productive firms are more likely to conduct R&D (e.g., Doraszelski and Jaumandreu, 2013). The assumption of exogenous productivity evolution will attribute the impact from R&D to the high persistence of productivity. I abstract from the endogenous productivity evolution in this paper due to the lack of R&D data.

[6]I abstract from the decision on enter or exit production, and assume that each firm in production always serve the domestic market. In data, I observe the production of all firms in the domestic market. Importantly, I observe who potential entrants (non-exporting firms) are and how they perform in the domestic market, in addition to exporting firms. So I do not need to assume an exogenous mass of potential entrants to the exporting markets whose information is not observed before their arrival as in Chaney (2008). Tracking productivity evolution and the learning process for all firms (including non-exporter/potential entrants of the foreign markets) is crucial to identify the roles of the two processes.

specific and unknown to the firm. As a result, the firm makes the export decision based on its productivity as well as its current demand belief in each foreign market at the beginning of period t (in addition to other relevant state variables).

I assume that the firm's export decisions for different markets are independent.[7] If the firm decides to export to market j, then it pays trade cost and receives (fulfills) n_{it}^{j} orders from customers in that market. By comparing the size of each order with its domestic sale, the firm can learn about how its demand factor in market j differs from its domestic demand, which is observed by the firm. In particular, the belief is updated at the end of period t according to Bayes' rule, based on these observations. However, if the firm decides not to export in market j, then it does not pay the trade cost, and there is no export profit generated from that market. More importantly, the belief regarding the demand factor in that market will not be updated at the end of period t because no transaction has happened (or been observed).

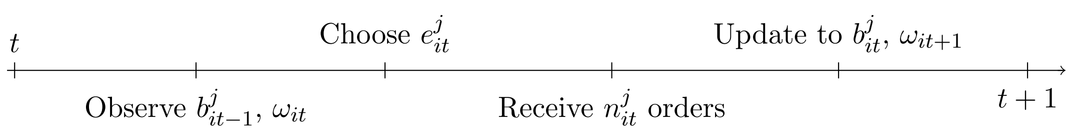
Several points need to be clarified. First, the firm's learning processes in different markets are assumed to be independent. It does not mean that underlying demand in different markets is independent. Instead, it assumes the firm does not apply what it observed in one market to another market. Some papers relax this assumption, but they either consider a finite-horizon model (e.g., Albornoz, Calvo Pardo, Corcos, and Ornelas, 2012), or one-shot learning (e.g., Nguyen, 2012). Also, they neither estimate a dynamic model nor take the effect of productivity into account, both of which are important to quantify the endogenous dynamic relationship between export participation and demand learning. If one is willing to assume that firms are learning about the demand at a global scope (i.e., a firm-level demand component which is common to all countries), then the identification and estimation methodology of this paper still carries over.[8] However, since I focus on how demand learning and productivity evolution drive the firm-market level exports, the independence of learning is assumed in this paper. Second, it is feasible to consider that productivity evolution is affected by export participation (i.e., learning by exporting); however, it is reasonable to abstract from this effect in this paper, because export participation in an *individual*

[7]Independence of export decisions does not necessarily imply the export participation outcomes are independent. Actually, the outcomes are correlated if productivity of manufacturing goods for different markets is the same within the firm. Empirically, I model the trade costs of exporting in a specific market as random variables to control for the possible trade-cost saving effect coming from the export status of the firm in other markets.

[8]An alternative direction is to adopt a more complicated framework of correlated learning as in Marcoul and Weninger (2008).

market (Germany, in this case) is plausibly having an insignificant effect on the *overall* evolution process of productivity.[9] Finally, the notion of the number of orders is carried over from the feature of the data (i.e., multiple transactions in a year to an individual market are commonly observed), but more importantly, it is crucial in the framework with demand uncertainty. In general, since export sales in a market are noisy signals of the true demand of the firm, the firm may exit the market if the signals are sufficiently bad. But how quickly the firm exits depends on how much of the information regarding the true demand has accumulated. In particular, firms with more orders (thus more signals) learn faster thus are more likely to exit with other factors being equal, as shown in the regression result in Section 4. Also, firms do not arbitrarily split orders into different shipments due to shipping costs.[10] Instead, the order intensity is found to be quite persistent as shown in Section 5.2. Thus, the notion of the number of orders is a natural observation of the data, and I explicitly model the order persistence with the purpose of taking the heterogeneous learning speed into account.

The time line of the model regarding the two key evolution processes is summarized as follows:



where ω_{it} is firm i's productivity in period t, b^j_{it-1} represents the belief about the demand factor in market j at the beginning of period t (or at the end of period $t-1$), and $e^j_{it} = 0$ or 1 represents the export participation in market j in period t.

In what follows, I describe the components of the model in detail by considering the firm's static decisions (on production and pricing), dynamic decisions (on exporting), as well as the evolution of the firm's belief about foreign demand.

[9] In a model with an AR(1) process of productivity evolution and the possibility of productivity gain from exporting: $\omega_{it} = g_0 + g_1\omega_{it-1} + g_2 e_{it-1} + \epsilon_{it}$, where e_{it-1} is an indicator of export decision of last period in Germany, I find that g_2 is insignificant.

[10] If firms split different shipments in order to learn the demand faster, then we would expect a larger number of order for new entrants of the foreign market than experienced exporters. But in data, the opposite is observed: experienced exporters receive 88.1% more orders.

2.2 Static Decisions

The firm's static decision is to set prices for the domestic market and foreign markets (where it decides to export) so that it maximizes the current profit in each period, after observing its productivity, capital stock, domestic demand, and aggregate demand in relevant markets.

2.2.1 Cost of Production

Each firm i faces a constant short-run marginal cost to produce its product for the domestic and foreign markets. I follow Aw, Roberts, and Xu (2011) to assume the logarithm of the marginal cost for period t as:

$$\ln c_{it} = \gamma_0 + \gamma_k \ln k_{it} + \gamma_w \ln w_t - \omega_{it}, \tag{1}$$

where k_{it} and ω_{it} are capital stock and physical productivity of firm i in period t respectively, and w_t is the wage rate common to all firms. Productivity and the marginal cost are the same to produce goods for all markets within a firm and a period, but they vary across firms and over time. The variation is captured by two sources of heterogeneity. The first one is the observable heterogeneity of k_{it}, which is exogenously given in each period.[11] The second source is unobservable (to researchers) firm-time specific productivity ω_{it}, which is assumed to evolve according to an exogenous first-order Markov process:[12]

$$\omega_{it} = g(\omega_{it-1}) + \epsilon_{it}, \tag{2}$$

where ϵ_{it} is an innovation term that is independently drawn from $N(0, \sigma_\epsilon)$. Productivity is observed by the firm, but is not observable by researchers.

Note that this cost specification takes the cost-side heterogeneity, both across firms and over time, into account. This is important in order to quantify how the export dynamics depends on the demand learning process, because otherwise the effects from cost-side variations may be attributed to the role of the learning process.

[11] In reality capital stock may evolve endogenously: more productive firms may choose to invest in capital and reduce their marginal costs. This leads to firms with high productivity and capital more likely to export. But in the empirical application, I consider a relatively short period (six years) thus follow the literature (e.g., Aw, Roberts, and Xu (2011)) to assume the firm has perfect foresight about it and treat the capital with a degenerate transition (fixed) in the dynamic problem.

[12] Compared with the random walk process assumed in Eaton, Eslava, Jinkins, Krizan, and Tybout (2013), this first-order Markov process allows for persistence which can be estimated from the data.

2.2.2 Demand and Pricing

The domestic market is assumed to be monopolistically competitive. In particular, firm i faces the Dixit-Stiglitz type demand curve in each period t:

$$q_{it}^D = \phi_t^D (p_{it}^D)^{\eta^D} e^{z_{it}^D}, \tag{3}$$

where $\eta^D < -1$ is the demand elasticity, and $z_{it}^D = z_i^D + \nu_{it}^D$ is the domestic demand shock. Here z_i^D measures the time-invariant demand specific to firm i that is not captured by the aggregate demand shifter ϕ_t^D. It includes all sources of firm demand heterogeneity that are unique to the domestic market. I refer to z_i^D as the domestic demand factor, which is a constant over time for firm i but is different across firms. ν_{it}^D is a random shock drawn from a mean-zero normal distribution. In data, given that most firms have operated in the domestic market for years, it is plausible to assume that firms observe their domestic demand z_i^D. In addition, when the revenue is realized, the firm can separately observe ν_{it}^D as well.

After observing its marginal cost, domestic demand and aggregate shifter, the firm sets a price to maximize its profit in the domestic market in period t:

$$\max_{p_{it}^D} \phi_t^D (p_{it}^D)^{\eta^D} (p_{it}^D - c_{it}) e^{z_{it}^D}.$$

The first order condition implies

$$p_{it}^D = \frac{\eta^D}{1+\eta^D} c_{it}.$$

Thus the domestic revenue for period t is

$$R_{it}^D = \phi_t^D \left[\frac{\eta^D}{\eta^D + 1}\right]^{\eta^D+1} \left[e^{\gamma_0} k_{it}^{\gamma_k} w_t^{\gamma_w} e^{-\omega_{it} + \frac{1}{\eta^D+1} z_{it}^D}\right]^{\eta^D+1}. \tag{4}$$

Notice that, as firm-level output prices in the domestic market are not observable in the data, productivity inverted from domestic sales is essentially revenue productivity: a combination of physical productivity which influences production cost directly and the demand shock that shifts the domestic market revenue at the firm level. Define the revenue productivity (based on the domestic revenue) as

$$\tilde{\omega}_{it} = \omega_{it} - \frac{1}{\eta^D + 1} z_{it}^D, \tag{5}$$

so the domestic revenue can be written in logarithm as:

$$\ln R_{it}^D = (\eta^D + 1)\ln(\frac{\eta^D}{\eta^D + 1}) + \ln \phi_t^D + (\eta^D + 1)(\gamma_0 + \gamma_k \ln k_{it} + \gamma_w \ln w_t - \tilde{\omega}_{it}). \tag{6}$$

That is, the domestic revenue is a function of demand elasticity, domestic aggregate demand shifter, wage rate, capital stock, and revenue productivity. I refer domestic-revenue-based productivity as productivity unless otherwise mentioned in the remain of the paper for simplicity, but keep in mind that it contains both physical productivity and domestic demand shocks due to the nature of the empirical way to recovering it. However, as shown later, this is what is needed for the empirical analysis from the point of view of researchers. Of course, in the firm's point of view, both physical productivity and revenue productivity are observed.

For each foreign market j, the firm's demand is heterogeneous in two aspects. First, firms are different in customer base and the sizes of distribution networks. Consequently, assuming that each order is from an unique buyer, this implies different order intensity: conditional on exporting, the expected number of orders in a period varies across firms. This heterogeneity does not only affect the profitability (more orders in a period means higher profit, with everything else being equal) but also influences the speed of demand learning (more orders implies more uncertainty is resolved in a period). If firm i decides to export to market j, then the number of orders it will receive in period t, n_{it}^j, is modelled as an random draw from a truncated Poisson distribution with parameter λ_{it}^j.[13] Thus, the expected number of orders is $\lambda_{it}^j/(1-\exp(-\lambda_{it}^j))$. In data, the number of orders received is different across firms but is persistent over time as documented in Section 4. Firms with a large number of orders last year tend to secure many orders in this year as well. To capture this, I allow the Poisson parameter to evolve over time at the firm level:

$$\ln \lambda_{it}^j = \psi^j(n_{it-1}) = \psi_0^j + \psi_n^j \ln(n_{it-1}^j + 1). \tag{7}$$

[13]The truncated Poisson is used since I assume if the firm exports it will receive at least one order. When trade costs are random, as I assume in the model, one cannot tell whether a firm attempted but no order was received or it simply decided not to export because of a high trade cost shock.

Here n^j_{it-1} is the number of transactions in period $t-1$ in market j. ψ^j_n measures the persistence of the number of orders. λ^j_{it} is different across firms and over time but is known by the firm when it makes its export decision. Note that this order process simply models the persistence of order intensity that shows up in data rather than imposing any restriction on which kind of firms are more likely to have more orders. Also, it is important to point out that product price does not play any role here: intuitively, when individual orders are from unique buyers, lowering price only influences the size of a given order rather than attracting more orders. This shares the same underlying assumption in Arkolakis, Papageorgiou, and Timoshenko (2015) and Berman, Rebeyrol, and Vicard (2015) that the price shifts the sale along the demand curve rather than the demand curve itself.

The second aspect of demand heterogeneity resides in the location of the demand curve: the quantity demanded at a given price in an individual order. With a slight abuse of the notation, I use n as the index of orders (or transactions) of firm i in market j and period t. For each firm i, as in the domestic market, the quantity sold in each order n in market j is endogenously determined by a price p^j_{in}, an aggregate demand shifter ϕ^j_t, and an idiosyncratic demand shock which can be decomposed as $z^j_{in} = z^j_i + \nu^j_{in}$. In the same spirit as in the domestic market, z^j_i measures the time-invariant demand specific to firm i, and I refer to z^j_i as the demand factor in market j, which is different across firms. ν^j_{in} is an unexpected idiosyncratic demand shock associated with order n and is i.i.d. drawn from a mean-zero normal distribution. Formally, the quantity demanded in transaction n is given by the demand curve:[14]

$$q^j_{in} = \phi^j_t (p^j_{in})^{\eta^j} e^{z^j_{in}}, \tag{8}$$

where $\eta^j < -1$ is the demand elasticity in market j, and ϕ^j_t is the aggregate foreign demand shifter (such as the market size etc.) as a counterpart in (3).

Conditional on exporting, the firm's profit maximization problem for each specific transaction n is to set a price that maximizes the profit in this transaction. At the beginning of period t,

[14] Note that this implies an analogy of *aggregate* expected demand curve as in the domestic market – the total quantity demanded from the firm is the sum of quantity over all orders: $E(q^j_{it}) = \lambda^j_{it}/(1-\exp(-\lambda^j_{it}))\phi^j_t (p^j_{it})^{\eta^j} E(e^{z^j_{in}}) \equiv \bar{\phi}^j_{it}(p^j_{it})^{\eta^j}$, where the firm's heterogeneity in total foreign demand from market j in period t, i.e. $\bar{\phi}^j_{it}$, consists of $\lambda^j_{it}/(1-\exp(-\lambda^j_{it}))$, $E(e^{z^j_{in}})$ and ϕ^j_t. That is, both the demand factor and order intensity influence the profitability in the foreign market.

the firm has observed its marginal cost c_{it} and the aggregate demand shifter ϕ_t^j. Thus, the profit maximization problem for transaction n is

$$\max_{p_{in}^j} \phi_t^j (p_{in}^j)^{\eta^j} (p_{in}^j - c_{it}) e^{z_{in}^j}. \tag{9}$$

Note that although the firm has not observed the demand shock z_{in}^j yet (when it makes decision on exporting or not), it does not affect the optimal pricing rule, but it does influence the expected quantity demanded and consequently the expected profit in this transaction. This implies, in contrast to the active pricing/experimentation in Balvers and Cosimano (1990) and Bergemann and Välimäki (2000), the period profit maximization is static. Thus, the first order condition implies:

$$p_{in}^j = \frac{\eta^j}{1+\eta^j} c_{it}. \tag{10}$$

Plug this equation into (9), utilize the marginal cost (1) and the revenue productivity defined in (5), then the revenue of transaction n can be written as:

$$r_{in}^j = \phi_t^j \left[\frac{\eta^j}{\eta^j+1}\right]^{\eta^j+1} \left[e^{\gamma_0} k_{it}^{\gamma_k} w_t^{\gamma_w} e^{-\tilde{\omega}_{it}}\right]^{\eta^j+1} \exp\left(z_{in}^j - \frac{\eta^j+1}{\eta^D+1} z_{it}^D\right). \tag{11}$$

Note that I use little case r_{in}^j to denote the revenue in individual orders to distinguish from total domestic revenue in a period, which is denoted as R_{it}^D.

Define

$$\zeta_{in}^j = z_{in}^j - \frac{\eta^j+1}{\eta^D+1} z_{it}^D = \overbrace{(z_i^j - \frac{\eta^j+1}{\eta^D+1} z_i^D)}^{\xi_i^j} + \overbrace{(\nu_{in}^j - \frac{\eta^j+1}{\eta^D+1} \nu_{it}^D)}^{u_{in}^j} \equiv \xi_i^j + u_{in}^j \tag{12}$$

as the difference between the demand shocks in the domestic market and the export market j adjusted by the demand elasticities, where ξ_i^j is the adjusted difference of the demand factors in the two markets and u_{in}^j is the adjusted difference of the idiosyncratic shocks. Note that ξ_i^j is still a constant and u_{in}^j has a normal distribution. Denote the standard variance of u_{in}^j as σ_u. This definition is useful because it allows me to express the export revenue in (11) in terms of (domestic-sale-based) revenue productivity $\tilde{\omega}_{it}$ and the demand difference ζ_{in}^j, in additional to other observable variables.

The demand and cost structure also imply a simple link between the profit of an individual transaction to its revenue:

$$\pi_{in}^j = -\frac{1}{\eta^j} r_{in}^j.$$

Consequently, the expected total export profit for the entire period t in market j is the sum of profit generated by all transactions in period t:

$$\begin{aligned}
E(\Pi_{it}^j) &= E_{n_{it}^j, \zeta_{in}^j}\Big(\sum_{n=1}^{n_{it}^j} \pi_{in}^j\Big) \\
&= E_{n_{it}^j, \zeta_{in}^j}\left(\sum_{n=1}^{n_{it}^j} \phi_t^j \frac{-1}{1+\eta^j}\left[\frac{\eta^j}{1+\eta^j}\right]^{\eta^j}\left[e^{\gamma_0} k_{it}^{\gamma_k} w_t^{\gamma_w} e^{-\tilde{\omega}_{it}}\right]^{1+\eta^j} e^{\zeta_{in}^j}\right) \\
&= \frac{\lambda_{it}^j}{1-\exp(-\lambda_{it}^j)} \phi_t^j \frac{-1}{1+\eta^j}\left[\frac{\eta^j}{1+\eta^j}\right]^{\eta^j}\left[e^{\gamma_0} k_{it}^{\gamma_k} w_t^{\gamma_w} e^{-\tilde{\omega}_{it}}\right]^{1+\eta^j} e^{\sigma_u^2/2} E(e^{\xi_i^j}).
\end{aligned} \tag{13}$$

The last equation holds because $E(e^{\zeta_{in}^j}) = e^{\sigma_u^2/2}E(e^{\xi_i^j})$ and the expected number of orders is $\lambda_{it}^j/(1-\exp(-\lambda_{it}^j))$. Therefore, the expected total export profit in market j depends on the aggregate demand shifter ϕ_t^j, capital stock k_{it}, wage rate w_t, revenue productivity $\tilde{\omega}_{it}$, the Poisson parameter λ_{it}^j, and the expectation of the demand factor difference $E(e^{\xi_i^j})$.

From (13), it is clear how revenue productivity (rather than physical productivity) and the demand difference come to directly affect the expected export profit (consequently, export propensity): revenue productivity influences the export profit by giving the firm a reference foreign profitability based on firm performance in the domestic market; the expectation regarding the demand difference affects export profit by adjusting the reference towards the true foreign profitability. Also, notice that although the aggregate demand shifter ϕ_t^j influences the profit (and consequently the propensity of exporting), it is observed by the firm and is taken into account when it makes the export decision.

However, the firm *may* face uncertainty about the foreign demand factor z_i^j. Because the domestic demand factor z_i^D is observed by the firm, the uncertainty regarding the level of the foreign demand factor z_i^j is, in the firm's point of view, equivalent to the uncertainty regarding the difference of domestic and foreign demand factors ξ_i^j. Thus, I can model the demand learning as a process of learning about the relative difference between the firm's domestic demand and foreign demand ξ_i^j. As shown in (13), this transformation enables me to utilize revenue productivity directly in modelling the export decision without recovering the physical production efficiency which requires either the output price data or a strong no-domestic-demand-heterogeneity assumption.

As a result, it is the belief about ξ_i^j which determines $E(e^{\xi_i^j})$ that *directly* influences the expectation of export profit and consequently the export propensity in market j, conditional on productivity $\tilde{\omega}_{it}$ and other variables. Nonetheless, the firm can learn about the true value of ξ_i^j from its own exporting experience: the firm observes the revenue in each order *after* exporting, and the comparison between the size of the order and the domestic revenue serves as a noisy signal regarding ξ_i^j. Take the power of $\frac{1+\eta^j}{1+\eta^D}$ to the domestic revenue (4), and take the ratio of that to (11), it is straightforward to show that the adjusted difference between the revenues in the domestic and exporting transactions reflects a realization of ζ_{in}^j:

$$\zeta_{in}^j = \ln r_{in}^j - \left(\frac{1+\eta^j}{1+\eta^D}\right) \ln R_{it}^D - \ln \Phi_t^j, \tag{14}$$

where $\Phi_t^j = \phi_t^j [\phi_t^D]^{-\frac{1+\eta^j}{1+\eta^D}} \left(\frac{\eta^j}{1+\eta^j} \frac{1+\eta^D}{\eta^D}\right)^{\eta^j+1}$. That is, (r_{in}^j, R_{it}^D) contains information about ζ_{in}^j. Since ζ_{in}^j can be viewed as a signal of ξ_i^j, the firm is able to learn about its underlying ξ_i^j from these signals and updates its belief. As a result, the belief regarding ξ_i^j evolves as the firm exports, and this comes to affect the expected value of exporting in market j in the future. The next subsection explains this learning process in greater detail.

2.3 Demand Uncertainty and Bayesian Learning by Exporting

In this section, I characterize uncertainty and Bayesian learning about foreign demand by exporting in a single market j. Since I assume the learning processes in different markets are independent, I omit the superscript for market j in order to simplify the notation.

I start to model the learning process by specifying firms' knowns and unknowns. The objective of the learning process is the demand difference ξ_i (or, as previously discussed, the level of foreign demand z_i equivalently), which is unknown by the firm.[15] However, the firm knows the distribution

[15]Another way to introduce the demand uncertainty is to assume λ_{it} is unknown and to be learned by the firm. However, the variance of the number of orders is large and suggests a learning speed which is too slow to be consistent with the data.

of ξ for the entire industry in that market, and this serves as a prior belief about its own ξ_i. More generally, I allow for observable heterogeneity in the prior belief across firms and model it as a function of firm-market characteristics. For example, firms may believe the demand is higher in markets with larger populations; also, some firms may have operated in the foreign market for a long time and have learned a lot (thus have smaller uncertainty) before I observe them in the data set. Formally, each firm holds a (known) prior belief $\xi_i \sim N(m_{i0}, \sigma_{i0})$ at the beginning of the initial period.[16] $m_{i0} = h_m(x_{i0}, z)$ is the prior expectation of the demand factor ξ_i while $\sigma_{i0} = h_\sigma(x_{i0}, z)$ captures the initial uncertainty, where x_{i0} is the firm's characteristics, such as age, ownership and firm size,[17] and z is the foreign market's characteristics, such as population and GDP.

The firm can learn the true value of ξ_i by observing ζ_{in} in each of its transactions. In particular, firm i observes ζ_{in} as a signal of ξ_i after exporting in period t. Note that $\zeta_{in} = \xi_i + u_{in}$, but ξ_i and the unexpected idiosyncratic demand shock u_{in} are not separately observed. For example, the firm cannot tell an big export sale is due to a persistent high demand or temporarily being lucky. Thus, the value of ξ_i is not immediately revealed because of the noise u_{in}. However, the firm knows the distribution of the noise: $u_{in} \sim N(0, \sigma_u)$. This knowledge enables the firm to update its belief according to Bayes' rule after observing a series of signals.[18] The standard deviation σ_u influences the speed of learning. In the extreme case where $\sigma_u = 0$, ξ_i can be accurately revealed after just one transaction. However, if σ_u is large, then the firm needs more signals to achieve a given level of accuracy. To this end, given σ_u, firms with more orders per period have a higher speed of uncovering their underlying foreign demand. In order to take the heterogeneity of learning speed into account, I explicitly model the order process (7) into the learning process.

More specifically, given the belief at the beginning of period t as $\xi_i \sim N(m_{it-1}, \sigma_{it-1})$, if firm i has decided not to export in period t, then the firm will not observe any new signals of its demand factor and there is no change in the belief at the end of period t. However, if the firm has decided to export and n_{it} orders are fulfilled in period t, then the belief will be updated after receiving n_{it} pieces of information $\{\zeta_{i1}, \zeta_{i2}, \ldots, \zeta_{in_{it}}\}$ in that period. Consequently, the posterior belief *at the*

[16] Empirically, "the initial period" means the first period when a firm appears in the data set. It is not necessary to be the period of its first export since firms' domestic production (thus existence) is observable.

[17] In particular, if productivity and the demand difference are correlated (e.g., firms with higher productivity expect their foreign demand to be even higher than their domestic demand), then the firm size can take this into account.

[18] I can also allow the firm to update the belief after each transaction, but since the firm only makes export decision in the beginning of each period, the two specifications are equivalent.

end of period t is given by $\xi_i \sim N(m_{it}, \sigma_{it})$, where[19]

$$m_{it} = \begin{cases} \dfrac{\sigma_u^2 m_{it-1} + \sigma_{it-1}^2 \tilde{\zeta}_{it}}{\sigma_u^2 + \sigma_{it-1}^2 n_{it}}, & \text{if exported in period } t \\ m_{it-1}, & \text{otherwise} \end{cases} \tag{15}$$

and

$$\sigma_{it}^2 = \begin{cases} \dfrac{\sigma_{it-1}^2 \sigma_u^2}{\sigma_u^2 + \sigma_{it-1}^2 n_{it}}, & \text{if exported in period } t \\ \sigma_{it-1}^2, & \text{otherwise} \end{cases} \tag{16}$$

and

$$\tilde{\zeta}_{it} = \sum_{n=1}^{n_{it}} \zeta_{in}.$$

Alternatively, the above equations can be written in terms of the initial belief and the entire history of signals received until period t: $\{\zeta_{i1}, \zeta_{i2}, \ldots, \zeta_{iN_{it}}\}$ where $N_{it} = \sum_{\tau=1}^{t} n_{i\tau}$:

$$m_{it} = \frac{\sigma_u^2 m_{i0} + \sigma_{i0}^2 \bar{\zeta}_{it}}{\sigma_u^2 + N_{it}\sigma_{i0}^2}, \tag{17}$$

and

$$\sigma_{it}^2 = \frac{\sigma_{i0}^2 \sigma_u^2}{\sigma_u^2 + N_{it}\sigma_{i0}^2}, \tag{18}$$

and

$$\bar{\zeta}_{it} = \sum_{n=1}^{N_{it}} \zeta_{in}.$$

Note that, in each period t, the belief about ξ_i is characterized by two variables: the mean and the standard deviation. The mean represents the expectation of ξ_i and may fluctuate over time depending on the entire history of signals that the firm received. However, the standard deviation, which measures the magnitude of uncertainty, is strictly decreasing as the firm receives more signals. As a result, for some firms, the expectation will increase if they keep observing large sales relative to their domestic sales, but for others, the expectation may decrease if their export sales are at levels lower than they expected. For all firms, however, standard deviations will keep falling if the firms keep exporting, which implies that uncertainty is resolving over time as the firms

[19] See DeGroot (2005), Chapter 9.

receive more signals.

2.4 Dynamic Decision – Export Participation with Demand Learning

In this section, I characterize a forward-looking firm's export participation with learning about its demand in an individual market j. Again, I omit the superscript for market j to simplify the notation. The term "export" or "not to export" means "export" or "not to export" to a *specific* market j. *Within* market j, the export decision and the learning process are endogenously related. The firm's export decision depends on its current belief regarding its foreign demand; moreover if the firm decides to export, then it expects the belief in the next period to be updated according to the signals received from exporting. Hence, due to this endogenous learning process, the export decision is dynamic. Moreover, the literature (e.g., Roberts and Tybout, 1997; Alessandria and Choi, 2007; Das, Roberts, and Tybout, 2007) has documented the importance of trade cost on rationalizing the trade participation persistence. I assume the trade cost consists of a deterministic part and a random part as follows:[20]

$$C(e_{it-1}, e_{it}) = e_{it-1}e_{it}c^f + (1 - e_{it-1})e_{it}c^s - \delta\epsilon(e_{it}), \tag{19}$$

where c^f and c^s are constant fixed cost and sunk cost respectively. Conditional on exporting, the firm pay a fix cost c^s if it exported in the last period; otherwise, it must pay a sunk cost c^f. The stochastic part is trade cost shocks $\epsilon(e_{it})$ with $e_{it} = \{0, 1\}$, both of which are i.i.d. draws from the standard type I extreme distribution with cumulative distribution function $G(\epsilon) = -\exp(-\exp(\epsilon))$. δ measures the scale of the shock. A larger δ implies a more dispersed trade cost distribution.

At the beginning of period t, given the current belief $N(m_{it-1}, \sigma_{it-1})$, the expected total export profit in period t (before considering the trade cost) is the sum of profit from all transactions in period t according to (13):

$$E\Big[\Pi(s_{it})\Big] = \frac{\lambda_{it}}{1 - \exp(-\lambda_{it})}\tilde{\phi}_{it}\exp(m_{it-1} + \sigma^2_{it-1}/2 + \sigma^2_u/2 - (\eta+1)\tilde{\omega}_{it}), \tag{20}$$

where $\tilde{\phi}_{it} = \phi_t \frac{-1}{1+\eta}\left[\frac{\eta}{1+\eta}\right]^{\eta} e^{\gamma_0(1+\eta)} w_t^{\gamma_w(1+\eta)} k_{it}^{\gamma_k(1+\eta)}$.

[20] A similar specification of cost is used in Barwick and Pathak (2015) to model the entry costs of housing agents.

To simplify notations, define the expected profit net of trade cost (given the firm exports) as:

$$u(s_{it}, e_{it-1}) = E[\Pi(s_{it})] - e_{it-1}c^f - (1 - e_{it-1})c^s \tag{21}$$

where e_{it-1} is the binary variable indicating the export status in period $t-1$. Summarize the set of state variables as $(s_{it}, e_{it-1}) \equiv (\tilde{\phi}_{it}, \lambda_{it}, \tilde{\omega}_{it}, m_{it-1}, \sigma_{it-1}, e_{it-1})$. Thus, the expected total export profit $E[\Pi(s_{it})]$ depends on the expected number of orders, productivity, the current demand belief, and a profit shifter $\tilde{\phi}_{it}$; whether or not the firm exported in the last period determines the fixed or the sunk cost to be paid. It is important to point out that the belief and productivity affect the expected profit via $\exp(m_{it-1} + \sigma^2_{it-1}/2)$ and $\exp(-(\eta+1)\tilde{\omega}_{it})$ respectively, thus this is how the two will be compared in empirical analysis of their individual roles.

Note that the current belief characterized by $(m_{it-1}, \sigma_{it-1})$ is a part of the state variables, since it affects the expected profit. More specifically, the expected profit is increasing in both the mean m_{it-1} and the standard deviation σ_{it-1}. In particular, this implies that, firms which face greater demand uncertainty are more likely to export, holding other variables fixed; likewise, firms have accumulated more signals have less uncertainty and are more likely to exit. This feature is an implication from (13) that profit is an increasing and convex function of foreign demand.

The timing of the entire model is summarized as follows:

1. At the beginning of period t, the firm observes (s_{it}, e_{it-1});

2. After observing its trade cost draw $C(e_{it-1}, e_{it})$, the firm decides whether to export or not (i.e., choose $e_{it} = 0$ or 1), based on its current state (s_{it}, e_{it-1}) which includes its current productivity and belief;

3. If the firm decides to export, then during period t, the firm pays the trade cost, receives and fulfils n_{it} orders from customers in the foreign market. The quantity and the price of each order are determined by (8) and (10) respectively. The firm is able to observe a signal ζ_{in} about ξ_i in each order by comparing the size of each order to its domestic sale at the end of this period. That is, a series of signals $\{\zeta_{i1}, \zeta_{i2}, \ldots, \zeta_{in_{it}}\}$ is observed, which is then used to update its belief about ξ_i. The posterior belief is $N(m_{it}, \sigma_{it})$, according to (15) and (16);

4. If the firm decides not to export, then there is no export profit from this market in this period; there is also no update to the belief at the end of this period (i.e., $m_{it} = m_{it-1}$ and $\sigma_{it} = \sigma_{it-1}$);

5. Period $t+1$ begins and all other state variables are updated.[21]

Given the timing, I model the firm's dynamic export participation using a Bellman equation. I denote the expected value function at the beginning of each period t before observing the trade cost shocks as $V(s_{it}, e_{it-1})$. The firm chooses to export to this market if the expected total firm value of exporting, after taking the trade cost (i.e., fixed cost or sunk cost) into account, is greater than firm value of not exporting. Thus, the Bellman equation is given by

$$
\begin{aligned}
&V(s_{it}, e_{it-1}) \\
&= E_{\epsilon_t^0, \epsilon_t^1} \max \begin{cases} \beta E[V(s_{it+1}, 0)|s_{it}, e_{it-1}] + \delta\epsilon(0), & \text{if} \quad e_{it} = 0 \\ u(s_{it}, e_{it-1}) + \delta\epsilon(1) + \beta E[V(s_{it+1}, 1)|s_{it}, e_{it-1}], & \text{if} \quad e_{it} = 1 \end{cases}
\end{aligned} \tag{22}
$$

where β is the discount rate, and

$$
\begin{aligned}
&E[V(s_{it+1}, e_{it})|s_{it}, e_{it-1}] \\
&= \int V(s_{it+1}, e_{it}) dF(s_{it+1}|s_{it}, e_{it-1}, e_{it}) \\
&= \int V(s_{it+1}, e_{it}) dF_\omega(\tilde{\omega}_{it+1}|\tilde{\omega}_{it}) dF_b(m_{it}, \sigma_{it}, \lambda_{it+1}|s_{it}, e_{it-1}, , e_{it}),
\end{aligned} \tag{23}
$$

and $F_\omega(\tilde{\omega}_{it+1}|\tilde{\omega}_{it})$ and $F_b(m_{it}, \sigma_{it}, \lambda_{it+1}|s_{it}, e_{it-1}, e_{it})$ are the transition probabilities of the four key state variables. It is important to note that the transition of productivity is independent from the export decision.[22] Thus there is no direct correlation between the two transition probabilities, $F_\omega(\cdot|\cdot)$ and $F_b(\cdot|\cdot)$. Nonetheless, the transition of the belief is affected by the export decision as well as the number of orders actually received in period t. I now turn to specify the transition probabilities of all four key state variables.

First, $F_\omega(\tilde{\omega}_{it+1}|\tilde{\omega}_{it})$ is given by the distribution of productivity in period $t+1$, conditional on the productivity in period t. Note that given the evolution (2), $\tilde{\omega}$ follows a first-order Markov

[21] The profit shifter $\tilde{\phi}_{it}$ is a combination of capital stock, wage rate, and the aggregate profit shifter. In the estimation, follow the literature (e.g., Aw, Roberts, and Xu (2011)) to treat $\tilde{\phi}_{it}$ as a fixed firm characteristic, while the remain state variables evolve as specified.

[22] If there is productivity gain from exporting, then the transition of productivity is related to the export decision. However, the methodology of the identification and estimation still follows.

process as well. Denote it as $\tilde{g}(\tilde{\omega}_{it})$. Thus, $\tilde{\omega}_{it+1}$ is drawn from $N(\tilde{g}(\tilde{\omega}_{it}), \sigma_\epsilon)$. That is,

$$F_\omega(\tilde{\omega}_{it+1}|\tilde{\omega}_{it}) = N(\tilde{g}(\tilde{\omega}_{it}), \sigma_\epsilon). \tag{24}$$

Second, $F_b(m_{it}, \sigma_{it}, \lambda_{it+1}|s_{it}, e_{it-1}, e_{it})$ is the joint distribution of the belief and λ_{it+1} at the beginning of period $t+1$, given the current state s_{it} and the export status (e_{it-1}, e_{it}). Note that the marginal distribution of (m_{it}, σ_{it}) and the distribution of λ_{it+1} are not independent, since both are related to the number of orders received in period t. Specifically, the joint probability, $F_b(m_{it}, \sigma_{it}, \lambda_{it+1}|s_{it}, e_{it-1}, e_{it})$, is determined according to (15) and (16), which is explained in detail as follows.

Given the belief at the beginning of period t as

$$\xi_i \sim N(m_{it-1}, \sigma_{it-1}),$$

the posterior belief at the end of period t (or, equivalently, the beginning of period $t+1$) remains the same if the firm has decided not to export in period t. That is, the transition probability is degenerate:

$$F_b(m_{it} = m_{it-1}, \sigma_{it} = \sigma_{it-1}, \lambda_{it+1}|s_{it}, e_{it-1}, e_{it} = 0) = 1,$$

where $\lambda_{it+1} = \exp(\psi_0 + \psi_n \ln(n_{it} + 1))$.

On the other hand, if the firm has decided to export, and suppose for the entire period t it receives a total of n_{it} signals $\{\zeta_{i1}, \zeta_{i2}, \ldots, \zeta_{in_{it}}\}$, then the updated belief is

$$\xi_i \sim N(m_{it}, \sigma_{it}),$$

where

$$m_{it} = m_{it-1} + n_{it}\frac{\sigma_{it}^2}{\sigma_u^2}\Big(\frac{1}{n_{it}}\sum_{n=1}^{n_{it}}(\xi_i + u_{in}) - m_{it-1}\Big) \tag{25}$$

and

$$\sigma_{it}^2 = \frac{\sigma_{it-1}^2\sigma_u^2}{n_{it}\sigma_{it-1}^2 + \sigma_u^2}. \tag{26}$$

Since m_{it-1} and σ_{it-1} are known at period t, the transition depends on random variables (n_{it}, ξ_i, u_{in}).

Note that the distributions of n_{it} and u_{in} are known, and ξ_i is believed to be distributed as the current belief, $N(m_{it-1}, \sigma_{it-1})$. Thus, *conditional on* n_{it}[23], the distribution of m_{it} is

$$m_{it} \sim N\Big(m_{it-1}, n_{it}\frac{\sigma_{it}^2}{\sigma_u^2}\sqrt{\sigma_u^2/n_{it} + \sigma_{it-1}^2}\Big) \equiv F_m(m_{it}|n_{it}, s_{it}, e_{it} = 1), \tag{27}$$

and the distribution of σ_{it} is degenerate:

$$F_\sigma\left(\sigma_{it} = \sqrt{\frac{\sigma_{it-1}^2\sigma_u^2}{n_{it}\sigma_{it-1}^2 + \sigma_u^2}}\middle| n_{it}, s_{it}, e_{it} = 1\right) = 1. \tag{28}$$

Since n_{it} is drawn from the truncated Poisson distribution with known parameter λ_{it}, the probability to receive n_{it} orders in period t is

$$F_n(n_{it}|s_{it}, e_{it-1}) = \frac{\lambda_{it}^{n_{it}} e^{-\lambda_{it}}}{n_{it}!(1 - e^{-\lambda_{it}})}. \tag{29}$$

Therefore, the joint transition probability of $(m_{it}, \sigma_{it}, \lambda_{it+1})$ is given by

$$\begin{aligned}&F_b(m_{it}, \sigma_{it}, \lambda_{it+1}|s_{it}, e_{it-1}, e_{it} = 1)\\&= F_m(m_{it}|n_{it}, s_{it}, e_{it} = 1)F_\sigma(\sigma_{it}|n_{it}, s_{it}, e_{it} = 1)F_n(n_{it} = \psi^{-1}(\lambda_{it+1})|s_{it}, e_{it-1}),\end{aligned} \tag{30}$$

where $\psi^{-1}(\cdot)$ is the inverse function of the order process (7).

These transition probabilities drive the firm's optimal decision on export participation according to the Bellman equation. In this way, I incorporate heterogeneity and evolution of both productivity and the demand belief into a model of exporting. In particular, (revenue) productivity evolves exogenously and influences the export decision by giving the firm a reference foreign profitability based on its performance in the domestic market; the demand belief evolves endogenously and it affects future export decisions by adjusting the reference towards the true foreign profitability based on the export outcomes. The next section demonstrates the strategy of identifying the effects of the two processes in firm export dynamics and estimating the structural model.

[23] According to the order process (7), the knowledge of n_{it} and λ_{it+1} is equivalent.

3 Identification and Estimation Strategy

The difficulty of the identification comes from the fact that both of the driving forces, productivity and belief updating, are heterogeneous across firms and evolve over time, and neither of them is observable to researchers. To identify the role of each force, I draw data from two sources. The first data source provides firm-level production information, which includes employment, labor and material expenditures, capital stock, domestic revenue, and other characteristics for each firm in each period. The second data source contains firm shipment-level exports, including the export destination, quantity and price of each shipment. The strategy for identification is to utilize that productivity affects both domestic revenue and export participation while the demand belief only influences export participation.

To be specific, a firm with higher physical productivity or domestic demand commands a higher domestic sale; but domestic sale itself does not depend on the evolution of the belief in any of the foreign markets. This relationship enables me to recover the a measure of time-varying revenue productivity for each firm in each period. In turn, I am able to estimate productivity evolution before considering the dynamic export decision.[24]

Indeed, the recovered revenue productivity is a combination of both physical productivity and the domestic demand shock. However, the size of the sale in each individual export transaction relative to the domestic sale implies a signal regarding how foreign demand *differs* from domestic demand. Consequently, export participation depends on both the revenue productivity and the demand belief. In order to recover the unobserved experience-dependent demand belief for each firm in a foreign market, I utilize the observed export participation together with the revenues of all individual transactions in that market. Specifically, the individual signals are used to recover the expectation of the belief; the number of transactions determines the resolution of uncertainty. Export participation depends on both the expectation and uncertainty of the belief, but in different ways. The probability of exporting is increasing in the expectation, which fluctuates over time depending on the observed sizes of individual transactions. However, since uncertainty is resolved over time as the firm exports, the export probability is decreasing in the number of transactions in a deterministic way, holding other factors fixed. Furthermore, a model with only productivity

[24] Allowing for productivity gain does not break the identification of productivity, but will make the empirical estimation of the structural model complicated.

heterogeneity predicts more productive firms export. While in my model with the two-dimensional heterogeneity, firms face greater demand uncertainty may also export even if their productivity is not high. Thus, with both productivity and demand beliefs being recovered, the cross-sectional and time series variations of export decisions identify the role of each driving force.

The estimation approach is inspired by the identification strategy. I divide the full set of parameters into a set of static parameters and a set of dynamic parameters. Before the dynamic estimation, I first estimate the static parameters, time-varying productivity, and demand signals received by each firm.

3.1 Estimation of Static Parameters, Productivity, and Demand Signals

As the first step, I estimate the set of static parameters: demand elasticity in the domestic market and foreign markets, and the marginal cost parameters. In addition, for each firm, I recover its time-varying productivity and demand signals received in each period. I use firm-level production data to estimate the marginal cost function and to recover productivity. Then I estimate the demand elasticities from the relationship between total variable costs and firm-market-level exports. Finally, I recover the transaction-level demand signals for each firm using shipment-level exports. These demand signals will be used to update firms demand beliefs in the dynamic estimation stage. The implementation of this strategy is specified as follows.

First, I estimate the marginal cost parameters and productivity using the firm-level production and domestic sale data. Specifically, the domestic revenue function (6) implies the observed domestic revenue is:

$$\ln R_{it}^D = (\eta^D + 1)\ln(\frac{\eta^D}{\eta^D + 1}) + \ln \phi_t^D + (\eta^D + 1)(\gamma_0 + \gamma_k \ln k_{it} + \gamma_w \ln w_t - \tilde{\omega}_{it}) + \tilde{\upsilon}_{it},$$

where $\tilde{\upsilon}_{it}$ is a measurement error. Note that the firm's productivity can be correlated with its capital stock. Thus, to control for the unobservable productivity, I follow Olley and Pakes (1996) and Levinsohn and Petrin (2003) to rewrite $\tilde{\omega}_{it}$ in terms of related observable variables. In general, firms' choice of variable material and labor inputs, m_{it} and ℓ_{it}, depends on the level of productivity and the demand beliefs about export markets. Since I assume that the marginal cost is constant in output, the relative input ratio is not a function of total output and thus does not depend on demand beliefs about export markets. Moreover, non-Hick-neutral technology implies that the differences in the mix of the two inputs across firms and over time reflect differences in productivity level.[25] Thus, I can write the unobserved productivity as a function of the relative input, conditional on the capital stock level: $\tilde{\omega}_{it} = \omega(k_{it}, m_{it}, \ell_{it})$. Then I combine the demand elasticity terms into an intercept $\tilde{\gamma}_0^D$ and use a set of time dummies, $\tilde{\Phi}_t^D = \phi_t^D w_t^{(1+\eta^D)\gamma_W}$, to capture the domestic industrial aggregate index as well as the wage rate which is common to all firms. Thus, the above equation can be written as:

$$\ln R_{it}^D = \tilde{\gamma}_0^D + \sum_{t=1}^{T-1} \gamma_t \tilde{\Phi}_t^D + f(k_{it}, m_{it}, \ell_{it}) + \tilde{v}_{it}, \tag{31}$$

where $f(k_{it}, m_{it}, \ell_{it}) = (1 + \eta^D)(\gamma_K \ln k_{it} - \omega(k_{it}, m_{it}, \ell_{it}))$ is a function of capital stock, material input and labor input. I parameterize function f as a cubic polynomial of these variables. Now the error term $\tilde{v}_{it}$ is uncorrelated with the right-hand-side variables. Thus, I use an ordinary least square regression to obtain the estimates. An important output from the regression is the fitted value of function f, which is denoted as $\hat{f}_{it}$, as an estimate of $(1 + \eta^D)(\gamma_K \ln k_{it} - \tilde{\omega}_{it})$. That is,

$$\hat{f}_{it} = (1 + \eta^D)(\gamma_K \ln k_{it} - \tilde{\omega}_{it}). \tag{32}$$

Then, I follow Olley and Pakes (1996) to construct a series of productivity measures for each firm, by utilizing the productivity evolution process. In particular, the first-order Markov process of productivity evolution is specified as

$$\tilde{\omega}_{it} = g_0 + g_1 \tilde{\omega}_{it-1} + \epsilon_{it}. \tag{33}$$

Substitute $\tilde{\omega}_{it} = -\frac{1}{\eta^D+1}\hat{f}_{it} + \gamma_K \ln k_{it}$ into the above evolution process and get

$$\hat{f}_{it} = -(\eta^D + 1)g_0 + g_1 \hat{f}_{it-1} + (\eta^D + 1)\gamma_K \ln k_{it} - g_1(\eta^D + 1)\gamma_K \ln k_{it-1} - (\eta^D + 1)\epsilon_{it}.$$

Again, the error term is uncorrelated with all right-hand-side variables. Thus, this equation can be estimated by nonlinear least squares estimation, since the function is nonlinear in g_1 and $(\eta^D+1)\gamma_K$. The key parameters estimated in this equation are $g_0^* = (\eta^D+1)g_0$, g_1, and $\gamma_K^* = (\eta^D+1)\gamma_K$. Note

[25] Non-Hicks neutral productivity has been found in a large empirical literature. See Stevenson (1980) for a model using plant-level data. Aw, Roberts, and Xu (2011) also utilize the same idea to recover productivity.

that η^D, g_0, and γ_K are not separately identified. However, if η^D is known, then $g_0 = \frac{g_0^*}{\eta^D+1}$ and $\gamma_K = \frac{\gamma_K^*}{\eta^D+1}$ are immediately recovered. More importantly, from (32), I can recover productivity as $\tilde{\omega}_{it} = -\frac{1}{\eta^D+1}\hat{f}_{it} + \gamma_K \ln k_{it}$ with knowledge of η^D.

To estimate η^D, I follow Aw, Roberts, and Xu (2011) and utilize the relationship between the total variable cost and domestic revenue as well as the total export revenue in each foreign market. Since the marginal cost of production is the same for domestic sales and exports, the first order conditions for profit maximization of the domestic and foreign markets imply that the total variable cost is a weighted combination of total revenue in each market. Specifically, for each firm i and each period t:

$$\begin{aligned} TVC_{it} &= q_{it}^D c_{it} + \sum_{j=1}^{J} q_{it}^j c_{it} \\ &= (1+\frac{1}{\eta^D}) q_{it}^D p_{it}^D + \sum_{j=1}^{J} (1+\frac{1}{\eta^j}) q_{it}^j p_{it}^j \\ &= (1+\frac{1}{\eta^D}) R_{it}^D + \sum_{j=1}^{J} (1+\frac{1}{\eta^j}) X_{it}^j, \end{aligned}$$

where TVC_{it} is the total variable cost, q_{it}^j is the total quantity exported to market j by firm i in period t, X_{it}^j is the corresponding total revenue, and R_{it}^D is the total revenue in the domestic market. Note that although the firm may export to market j with multiple transactions, the demand function (8) implies that the optimal price is proportional to the marginal cost, as shown in (10). Thus the second equality in the above equation holds. Therefore, the following empirical equation can be used to estimate η^D as well as all η^js:

$$TVC_{it} = (1+\frac{1}{\eta^D}) R_{it}^D + \sum_{j=1}^{J} (1+\frac{1}{\eta^j}) X_{it}^j + \upsilon_{it}, \tag{34}$$

where υ_{it} is a measurement error.

Hence, up to now I have obtained the key estimates $\hat{g}_0$, $\hat{g}_1$, $\hat{\gamma}_K$, $\hat{\eta}^D$, and all $\hat{\eta}^j$s. So, it is straightforward to recover the productivity for each firm in each period as $\tilde{\omega}_{it} = -\frac{1}{\hat{\eta}^D+1}\hat{f}_{it} + \hat{\gamma}_K \ln k_{it}$. In addition, with $\hat{\eta}^j$s, I can recover the aggregate demand shifter ϕ_t^j in the foreign demand function (8) using the relationship between shipment-level exports and domestic revenue (14) via an ordinary least squares regression:

$$\ln r_{in}^{j} - \left(\frac{1+\hat{\eta}^{j}}{1+\hat{\eta}^{D}}\right)\ln R_{it}^{D} = \ln \Phi_{t}^{j} + \xi_{i}^{j} + u_{in}^{j}, \tag{35}$$

Since the demand factor is uncorrelated with the aggregate demand shifter, the regression produces unbiased estimates of Φ_t^j which reveal the aggregate demand shifter $\hat{\phi}_t^j$ because all other constants have been estimated.[26] The regression also provides an estimate of the standard deviation of the noise, $\hat{\sigma}_u^j$, by taking the difference of (35) within firms to eliminate time-invariant ξ_i^j.

After obtaining these estimates, I can recover the demand signals received by firms as

$$\zeta_{in}^{j} = \ln r_{in}^{j} - \left(\frac{1+\hat{\eta}^{j}}{1+\hat{\eta}^{D}}\right)\ln R_{it}^{D} - \ln \hat{\Phi}_{t}^{j}. \tag{36}$$

Then I use demand signals to update firms' market-specific demand beliefs according to Bayes' rule as specified in (17) and (18).

To sum up, the static estimation stage provides the estimates of the marginal cost function parameters, time-varying productivity measure for each firm with the productivity evolution process as well as the market-specific demand signals received by each firm in each period. The demand signals enable me to write firms' demand beliefs as a function of initial beliefs up to a set of parameters to be estimated in the dynamic stage.

3.2 Estimation of Dynamic Parameters

The set of dynamic parameters includes the parameterized initial belief functions, $h_m(\cdot|\beta_m)$, $h_\sigma(\cdot|\beta_\sigma)$, the trade cost distribution parameters, and the parameters of the order process, ψ_0 and ψ_n in (7).

3.2.1 Estimation Details

I estimate the dynamic parameters via the Maximum Likelihood Method. The likelihood is constructed from the data on the discrete choice of export participation together with the number of transactions for each firm in each period. To simplify notation, I only consider the exports to a

[26]Note that as the estimated domestic demand shifter contains the wage rate which is common to all firms: $\tilde{\Phi}_t^D = \phi_t^D w_t^{(1+\eta^D)\gamma_W}$, so the foreign market demand shifter can only identified as $\tilde{\phi}_t^j = \phi_t^j w_t^{(1+\eta^j)\gamma_W}$. However, for the foreign market profit, $\tilde{\phi}_t^j$ is exactly what is needed.

single market j and omit the market superscript, since the export decisions and learning processes in different markets are independent for each firm. However, it is straightforward to extend the estimation to all foreign markets.

For each firm i in each period t, I observe the export participation decision e_{it}. If the firm exported in period t, then $e_{it} = 1$, and I can observe the number of transactions n_{it} in that period, as well as the revenue (r_{in}) in each transaction.[27] If the firm did not export to that market in period t, then $e_{it} = 0$ and no transaction happened.

The full likelihood consists of two partial likelihoods. The first one is about the number of transactions n_{it} of each firm i in each period t. The parameters involved include the parameters for the evolution process of λ_{it}, which is summarized in the vector $\theta_1 = (\psi_0, \psi_n)$. The first partial likelihood is given by the truncated Poisson probability since the number of transactions is assumed to be strictly greater than zero if the firm exports:

$$\ell^1(n_{it}; \theta_1) = \frac{\lambda_{it}^{n_{it}} e^{-\lambda_{it}}}{n_{it}!(1 - e^{-\lambda_{it}})},$$

where λ_{it} is specified as (7).

Given θ_1, the second partial likelihood is about the discrete choice of export participation e_{it}, conditional on (s_{it}, e_{it-1}). The parameters involved are parameters for initial beliefs and trade cost distribution parameters, which are summarized in the vector $\theta_2 = (\beta_m, \beta_\sigma, c^f, c^s, \delta)$. The second partial likelihood is given by:

$$\begin{aligned} &\ell^2(e_{it}|s_{it}, e_{it-1}; \theta_1, \theta_2) \\ &= e_{it} \Pr(e_{it} = 1|s_{it}, e_{it-1}; \theta_1, \theta_2) + (1 - e_{it}) \Pr(e_{it} = 0|s_{it}, e_{it-1}; \theta_1, \theta_2) \end{aligned} \tag{37}$$

where $\Pr(e_{it} = 1|s_{it}, e_{it-1}; \theta_1, \theta_2)$ is the conditional probability of exporting. It depends on the parameters θ_1 and θ_2. Given the standard type 1 extreme value distribution of the trade cost

[27] Note the superscript of market j is suppressed, and I use n as an index of transactions.

shock, the conditional probability of exporting is

$$
\begin{aligned}
&\Pr(e_{it} = 1 | s_{it}, e_{it-1}; \theta_1, \theta_2) \\
&= \Pr\Big(u(s_{it}, e_{it-1}) + \delta E[V(s_{it+1}, 1) | s_{it}, e_{it-1}] - \beta E[V(s_{it+1}, 0) | s_{it}, e_{it-1}] > \epsilon(0) - \epsilon(1)\Big) \\
&= \frac{\exp(u(s_{it}, e_{it-1}) + \beta E[V(s_{it+1}, 1) | s_{it}, e_{it-1}])}{\exp(u(s_{it}, e_{it-1}) + \beta E[V(s_{it+1}, 1) | s_{it}, e_{it-1}]) + \exp(\beta E[V(s_{it+1}, 0) | s_{it}, e_{it-1}]))},
\end{aligned}
\tag{38}
$$

where $u(s_{it}, e_{it-1})$ is the expected net export profit given by (21), and $E[V(s_{it+1}, e_{it}) | s_{it}, e_{it-1}]$ is the expected firm value at the beginning of period $t + 1$, as described in the Bellman equation. Note that the belief affects this probability through both $u(s_{it}, e_{it-1})$ and the expected future firm value $E[V(s_{it+1}, e_{it}) | s_{it}, e_{it-1}]$.

Thus, the full likelihood is the product of the two partial likelihoods:

$$
\ell^f(e_{it}, n_{it} | s_{it}, e_{it-1}; \theta_1, \theta_2) = \ell^1(n_{it}; \theta_1)\ell^2(e_{it} | s_{it}, e_{it-1}; \theta_1, \theta_2).
$$

I denote the full set of dynamic parameters as $\theta = (\theta_1, \theta_2)$. Then, θ can be estimated via maximizing the likelihood:

$$
\hat{\theta} = \arg\max_{\theta} \sum_{i,t} \ln(\ell^f(e_{it}, n_{it} | s_{it}, e_{it-1}; \theta_1, \theta_2)). \tag{39}
$$

However, it is computationally difficult to estimate θ in this way. This is because the evaluation of the full likelihood requires solving the unknown value function V implicitly defined by the functional Bellman equation (22). In this application, both the high dimensions of the state variables and the number of structural parameters reduce the ability to quickly compute the value function, which is crucial in the dynamic empirical model. To solve this issue, I reduce the computational burden in two ways. First, I adopt the strategy used by Rust (1987) to estimate the parameters by three stages, which is described in Appendix A. Second, I follow the pioneer practice in Barwick and Pathak (2015) to use Sieve functional approximation combined with MPEC developed in Su and Judd (2012) in order to evaluate the value function without solving it. The basic idea is to approximates the unknown value function using Sieves with parametric basis functions (Chen, 2007) so that I can cast the Bellman equation as a model constraint in the estimation procedure that has to be satisfied at the parameter estimates. This formulation avoids the need to solve the Bellman equation with iterations and makes the estimation of structural parameters in this high-dimension-state dynamic model possible.[28] The implementation in my application is specified in Appendix B.

4 Data

I draw data of ceramics and glass industry in China with exports to German from two sources to estimate the structural model and quantify how the two unobservable driving forces separately explain the firm-market-level export participation.

The first source is the Chinese Customs Transactions database, which includes all export shipments of Chinese firms from 2000 to 2006. It is common that firms have multiple shipments to an individual destination market within a year. For example, the median number of shipments of a firm in this industry to German in a year is 4 with a standard deviation of 6.1. The significant dispersion of transaction intensity suggests the importance to control for the learning speed across firms explicitly. Each shipment contains shipment value, quantity, 8-digit HS code, type of trade, destination market, shipment month, and firm identification number. Beyond what firm-level annual trade data commonly used in the literature provides, I can observe whether a firm exported to a market or not in each year; the number of transactions and the revenue of each transaction. The transaction-level revenue enables me to recover the demand signals; the variation of the export experience measured by the number of transactions that firms conducted contains information about how demand uncertainty resolves over time. Consequently, I use the change of export patterns over time to identify how firm-market-level export participation is endogenously related to heterogeneity in demand learning, with productivity controlled by the supplement of the second data source.

The second data source is the Chinese Annual Survey of Manufacturing from the Chinese National Bureau of Statistics. This database covers the period 2000 to 2006 and provides detailed annual firm-level production information of all medium and large manufacturing firms that had total annual sales of more than $600,000. The primary variables include firm-level domestic revenue, labor wage, employment, material input, capital stock, ownership, and firm age. I draw these

[28]The validity and properties of this methodology are discussed in Barwick and Pathak (2015).

variables for ceramics and glass industry from this database, construct a firm-time specific productivity measure, and estimate the marginal production cost function with observable cost shifters such as capital stock as specified in Section 3. A challenge of dealing with the data sources is that the sets of firm identification numbers are different in the two data sources since they are collected by different agencies. Thus, I match the two databases according to the recorded firm name, phone number, zip code, and other identifying variables.[29]

In this paper, I utilize the data of ceramics and glass industry for several purposes. First, firms in this industry produces sanitation, special, and daily-used ceramics and glass. The major products for exporting are colorful dinnerware and ornamental articles of ceramics such as statuettes, which can be viewed as a concentrated product line. The percentage of exporting firms is around 22% and the maximum export market share (within the Chinese firms) is 4.3% so that I can abstract from strategic interactions. Second, firms in this industry are likely to face demand uncertainty due to the nature of the export type. It is well-known that an important feature of Chinese exports is that a significant portion of transactions are processing trade, in which domestic firms' intermediate material and even relevant production technology are directly supplied by foreign partner firms and their final products are supposed to ship to foreign markets as instructed by their foreign partners. These firms are more like long-term contractors rather than initiative exporters, and their export decisions are less likely to be affected by the uncertainty in foreign markets. Unlike these firms, most of export transactions in the ceramics and glass industry are *ordinary trade*, in which firms make their own decisions on production, pricing, and exporting, without being constricted by the existing partnership with foreign companies. Thus, demand uncertainty may play an important role in their export decisions. In the estimation of static parameters, I use the firm-level production data for all firms in the ceramics and glass industry, and the estimation of dynamic parameters is based on the data of transaction-level exports from 1045 firms that can be identified from both data sources. I focus on the exports to Germany as a representative market, which is the fifth popular (rather than the most popular) destination by both total export volume and the total number of export transactions, so that these firms are likely to face demand uncertainty in Germany due to the

[29] For the two entire database, about 114,000 out of 278,000 firms in the custom database are matched (around 41%). Note that the custom data records all transactions for all firms, while the annual survey data only records medium and large manufacturing firms, thus the percentage of matched firms conditional on medium and large manufacturing firms should be larger than 41%.

geographic distance and potential different consumer tastes. Moreover, these firms are non-trivial firms as they are identified from both data sources only if they had conducted some international trade (to be identified from the Custom Transactions Database) and were above the $600,000 sale threshold (to be identified from the Annual Survey of Manufacturing). Also, they are unlikely to be intermediate (trading) firms, after careful inspection of the firm names and input and output ratios.

The integrated data set reveals dynamic features of firm-market-level export participation that are consistent with a model of uncertainty and Bayesian learning about foreign demand. First, a large percentage of firms drop out after the first year of exporting to a market, despite significant sunk cost of initiating exporting. I follow Eaton, Eslava, Jinkins, Krizan, and Tybout (2013) and consider the export cohorts that began exporting to the German market in a particular year. In Table 1, each column reports the percentages of firms within each cohort that chose to export after the year of entry. The pattern shows that there is a significant percentage (around 35%) of firms that drop out of the German market after the first year of exporting. However, conditional on survival in the second year, the percentage of exporters decreases over time but with a much smaller magnitude. Aggregate demand shocks or trade cost reduction cannot explain the large attrition rate after the first entry, since the high attrition appears in all cohorts that entered in different years (as shown in the different columns in Table 1) as well as in other destinations. Did it result from a negative firm-level productivity shock? Using firm-level production data, I find that, for firms that dropped out after the first year of exporting, 45% of them were experiencing decreases in a simple measure of labor productivity (measured as value added output per worker) while 55% of them were experiencing increases in labor productivity. Thus, although it is possible that supply side factors play a role, it is unlikely that they alone can tell the full story of the high attrition rate.

Second, the export decisions of new exporters *gradually* become stable over time. To ensure that the firms under consideration are more likely to be new exporters in the German market, I define new exporters as the firms that started to export to the German market in the last quarter of 2001 but did not export to the German prior to that. I calculate the percentage of these firms that switched their export status (from export to not to export, or vice versa) in each quarter after 2002. Figure 1 shows that overall this percentage becomes smaller over time. This implies that new exporters' export decisions become more stable over time. More importantly, the percentage *gradually* becomes stable after several continuing significant drops. This pattern is not limited to the German market or the firm cohort that began to export in a particular year, but is a universal feature that also appears in other export cohorts and in other destinations. Note that, potentially, a simpler model without Bayesian learning is to allow the one-shot learning and serially correlated idiosyncratic demand, which is observed by firms but is unobservable to researchers. However, such a model will predict the switching percentage to drop very shapely after the first entry and immediately become stable afterwards (because of completely resolved uncertainty), which is not consistent with the gradual stabilization in Figure 1.

Furthermore, the transactional level exports imply the number orders received by individual firms play an important role in firms' export decision in the circumstance of demand uncertainty. The firms with higher order intensity are not only in general more likely to export but also learn faster when they export, because they receive more signals. The implication is that firms that have accumulated more signals are more like to exit. As support to this implication, I run a simple linear probability model of post-entry decisions for the firms that entered to Germany during 2002-2004. I regress (post-entry) export participation in the German market against their order intensity and accumulative orders received in the end of the previous year. That is,

$$\Pr(e_{it} = 1) = \alpha_0 + \alpha_n \ln(n_{it-1} + 1) + \alpha_N \ln(N_{it-1} + 1).$$

Here n_{it-1} is the number of orders received in year $t-1$, measuring the order intensity in the coming year t (given its persistence); N_{it-1} is the *total* number of orders received up to year $t-1$, measuring the magnitude of uncertainty remain (a larger N is associated with a lower level of uncertainty). The estimates are summarized in in Table 2. $\hat{\alpha}_n$ is positive, suggesting that firms with higher order intensity are more likely to export. But more importantly, $\hat{\alpha}_N$ is estimated as negative. This implies that firms with more experience are more likely to exit the export market after controlling for the order intensity n. This suggests the necessity to explicitly control for the heterogeneity of the learning speed of individual firms in order to quantify the role of belief evolution in the export decisions.

Overall, the above patterns show that it is possible that both productivity evolution and

Bayesian learning about demand contribute to the observed export dynamics. But it remains an empirical question to investigate the magnitude of each mechanism in determining export dynamics. In order to identify and isolate individual roles of the two forces, now I turn to the estimates of the structural model parameters.

5 Empirical Results

5.1 Estimates of Static Parameters

The static parameters include the parameters in the marginal cost function (1), productivity evolution (2), and the demand elasticities in the domestic and foreign markets. The estimation uses the firm-level production data for all firms in the ceramics and glass industry.

Table 3 shows the estimates of the marginal cost and productivity parameters. Note that coefficient γ_k measures the elasticity of capital in the marginal cost. The negative estimate (-0.073) means larger firms have more cost advantage. More importantly, the high estimate $\hat{g}_1 = 0.844$ in the AR(1) productivity evolution implies that productivity evolves persistently. A large part, but not all, of productivity is carried over from one year to another. At the same time, the standard deviation of the innovation term, $\sigma_\epsilon = 0.179$, implies that there is a significant unexpected productivity shock that shifts firm productivity over time. This suggests that it is necessary to take the evolution of productivity into account in the investigation of the learning process.

Although the estimation focuses on the exports to Germany, as shown in Section 3.1, I need the estimate of domestic demand elasticity in order to recover firm-time productivity. Also, the demand elasticity in Germany is needed to construct demand signals. Thus, I utilize (34) by aggregating the other markets to a single market with elasticity η^{oth}, and estimate them together. This essentially simplifies the estimation equation (34) to

$$TVC_{it} = (1 + \frac{1}{\eta^D})R_{it}^D + (1 + \frac{1}{\eta^{DEU}})X_{it}^{DEU} + (1 + \frac{1}{\eta^{oth}})X_{it}^{oth} + \upsilon_{it},$$

where υ_{it} is the measurement error. The result reported in Table 4 indicates reasonable estimates of demand elasticities.

By estimating (35), I find the standard deviation of signal noise u_{it} for Germany is $\hat{\sigma}_u = 1.661$.

It measures the informativeness of each individual signal. If $\hat{\sigma}_u$ is high, then each transaction contains less effective information about the true value of ξ_i; accordingly, it takes the firm more transactions to reach a certain accuracy of the belief.

5.2 Estimates of Dynamic Parameters

The dynamic parameters include the parameterized initial belief and of the trade cost distribution parameters, and parameters for the order process. The estimation uses shipment-level export data from the Chinese ceramics and glass industry to Germany from 2000 to 2006, together with the estimates from the static stage.

It is worth to point out that the initial period is defined as the period that a firm first appears in the data set,[30] and the initial belief is the belief held by a firm in its initial period. Since the data set only includes exports from 2000, I do not observe firms' export status before that. Thus, I use the firm state variables in 2000 as an initial condition and allow firms to hold different initial beliefs. In principle, it is possible to allow the initial belief to be heterogeneous across firms. For example, firm age, capital stock and ownership are observable in the data set and I can write the initial belief as a function of these observable characteristics with parameters to be estimated. However, this dramatically increases the number of dynamic parameters to be estimated. In order to reduce the computational burden, I allow for the heterogeneity by dividing firms into two groups: potential entrants in the German market and experienced exporters according to their export status to Germany in their initial periods. A firm is defined as an experienced exporter if it exported to Germany in 2000 (or in the first year it appears in the data), otherwise it is defined as a potential entrant. It is worth to note that, although potential entrants did not export in the first year, they can choose to export in every subsequent year. I assume potential entrants and experienced exporters to have initial belief $N(m_{00}, \sigma_{00})$ and $N(m_{10}, \sigma_{10})$ respectively. The fixed and the sunk entry costs are assumed to be the same for the two groups.

The estimates of the order process parameters are given in Table 5. $\hat{\psi_n} = 0.706$ implies that the persistence of the order intensity is high, which is consistent with the observation in the data. Firms with more transactions in the last year are also likely to secure more transactions this year,

[30] Note that as long as a firm appears in the Manufacturing Survey data, it counts toward "first appearance in the data set" – it does not need to be its first export.

if they choose to export. Thus, it is important to take this into account in order to control for the different learning speeds across firms before quantifying the role of the learning process. The estimates of the trade cost parameters are reported in Table 5 as well. These estimates suggest the (unconditional) means of the fixed and sunk cost as 46.4 thousand USD and 273.1 thousand USD respectively. This reflects that the trade costs prevent all but only the productive firms and/or firms with high demand expectation from exporting.

The estimates of initial beliefs in Table 5 suggest that compared with potential entrants, experienced exporters tend to have higher initial expectations and face less uncertainty. This is reasonable, since unlike potential entrants, these firms may have operated in the foreign market for a long time and have learned a lot before 2000. Given that the estimate of standard deviation of signal noise $\hat{\sigma}_u = 1.661$ is significantly larger than the standard deviations of initial beliefs, Figure 2 shows that it takes 115 more transactions (signals) for a new exporter to reach the same accuracy (0.138) of the belief as an experienced exporter.

Export participation can be influenced by the learning process through two different channels. The first channel is the magnitude of uncertainty. The value difference between the two choices, to export and not to export, is decreasing and concave as demand uncertainty resolves, holding other factors constant. Since the export decision relies on the comparison between this value difference against the fixed or sunk cost that the firm incurs, the decreasing value difference suggests that firms have less incentive to export as more information is accumulated. This rationalizes the stylized fact that many firms quit exporting within a short period after entry. However, this does not necessarily mean that all firms are less likely to export over time, since their updated expectations vary according to the signals received. This is the second channel that the learning process takes effect: a high expectation implies a high expected value of exporting which encourages the firm to export. In this way, the demand belief, which consists of both the mean and standard deviation (as a measure of the magnitude of uncertainty), drives the export decision.

However, how productivity and the demand belief influence export participation separately? Table 6 compares the differences between exporting firms and non-exporting firms in these two dimensions. Recall that the expectation about the relative difference of the demand factors is defined as $\mathrm{E}(\exp(\xi)) = \exp(m + \sigma^2/2)$. The table shows that firms with high productivity and/or high demand expectation are self-selected to the exporting market. Considering productivity is measured from the domestic sales, the result suggests that the exporting firms are on average 3.8% more profitable in the domestic market than non-exporting firms,[31] which is driven by the difference in physical efficiency and/or domestic demand. However, on average, the foreign demand expectation (measured by $m + \sigma^2/2$) is higher for exporting firms than non-exporting firms by 0.455. This implies that exporting firms expect 57.6% (i.e., $\exp(0.455) - 1$) more demand than non-exporting firms, on top of their domestic demand. This comparison across productivity and demand belief suggests that both of them are indeed determinants of the export decision, but, interestingly, the main driving force resides in a much higher expectation regarding foreign demand rather than their superior performance in the domestic market.

Furthermore, the differences of the heterogeneity in productivity and the demand belief carry over in the comparison of potential entrants and experienced exporters. Figure 3 shows the kernel densities of productivity for potential entrants and experienced exporters respectively.[32] Although there is significant productivity heterogeneity within both groups, the densities almost overlap with each other. The difference of the means between the two groups is marginal: 0.019. This implies that on average experienced exporters are 2.4% more profitable in the domestic market than potential entrants. In contrast, Figure 4 shows the kernel densities of the demand expectation (measured as $m + \sigma^2/2$) for the two groups respectively. Again, the result indicates substantial firm heterogeneity in the demand expectation within each group. Consistent with their initial beliefs, experienced exporters hold more optimistic and less dispersed demand beliefs, compared with potential entrants. More importantly, the demand belief difference between the two groups is large. In particular, the difference of the mean foreign demand (m) reported in Table 7 suggests that, experienced firms on average expect 159.3% more demand compared with potential entrants (on top of their own domestic demand), while this is offset by the lower uncertainty (σ) they face in the market by 3.5%. The total effect is, on average experienced exporters expect around 1.4 times more profit resulted from high foreign demand expectation for an individual transaction (on top of their own domestic profitability difference, which is documented as marginal), compared with potential entrants.[33] This, together with 88.1% more orders received by experienced firms in a

[31]Note that productivity enters the period profitability function as $\exp(-(\eta+1)\omega)$, so the profitability difference is calculated as $\exp(-(\eta+1)\Delta\omega) - 1$.

[32]Productivity estimates across firms and over time are pooled together to obtain the densities. The level of productivity is normalized to a common reference point for comparison.

[33]Note that $E(\exp(\xi)) = \exp(m+\sigma^2/2)$ so the expected profitability difference is calculated as $\exp(\Delta m + \Delta\sigma^2/2) -$

year, implies that, in total experienced firms expect 3.8 times more profit in a year on average. Of course, the way of dividing groups, based on export status in 2000, naturally suggests that firms with higher productivity and/or higher demand beliefs are selected to be experienced exporters. The above comparison confirms it but reveals more than that: although experienced exporters are superior to potential entrants in both the foreign demand and productivity, the former is the dominant difference. Given the productivity actually reflects firm profitability in the domestic market, this finding is reasonable: by the nature of the sample, all firms are medium and large firms that have some international trade experience in foreign countries, so it is plausible that the two groups have similar distributions of domestic profitability and the main factor that distinguishes them as experience exporters is the high demand expectation in the German market. Nonetheless, considering the significant productivity dispersion *within* each group, the results show that although the domestic performance is positively correlated to export participation, it is the foreign demand that serves as the main predictor of the success in the German market.

It is interesting to notice that how the initial demand beliefs change to updated beliefs for both groups. Figure 5 shows that the updated beliefs become less dispersed around the center of the initial beliefs. This suggests two kinds of firms both exist: some optimistic firms which overestimated their demand became more realistic and adjusted their expectation to a lower level; some pessimistic firms which underestimated their demand became more positive and changed their expectation to a higher level. And this is true for both potential entrants and experienced firms, with the case of potential entrants being more obvious. Specifically, the mean posterior expectation (m_{it}) of potential entrants is higher than their initial belief by 4.3%. These results together provide evidence that, after receiving favorable signals, some entrants found their demand was higher than the initial belief, and they thus decided to continue exporting and survived in the German market. So if potential entrants are promoted to export because of favorable policy influence such as trade cost reduction, then some entrants may eventually survive in the foreign market if they have learned high demand. This directly motivates the counterfactuals in the next section to investigate the indirect effect of trade cost reduction on promoting export in the environment of demand uncertainty.

$1 = \exp((-0.570 + 1.523) + (0.132 - 0.296)/2) - 1 = 1.4.$

6 Counterfactual Analysis

The results in the previous section indicate substantial heterogeneity in both productivity and the demand belief, and show that both processes are driving forces of export dynamics. However, what is the relative importance of the two processes for different firms? What is the implication of trade liberalization (via trade cost reduction) in the circumstance of demand uncertainty? In this section, I employ the estimated model to explore how the two forces affect export participation separately for potential entrants and experienced exporters in the dynamic dimension. Specifically, I conduct two sets of counterfactual analysis. In the first exercise, I shut down either of the processes and re-solve the restricted dynamic model to evaluate how export participation is influenced by that process. This sheds light on the importance of each driving force in the dynamic dimension. In the second exercise, I resolve and simulate the firm's endogenous choice of export under a trade cost reduction policy in order to evaluate the role of the learning process in promoting export participation.

I consider two restricted scenarios in the first set of analysis. First, I shut down the learning process by assuming that each firm knows its demand factor and faces no uncertainty. Specifically, given the belief of each firm and each year, I remove the uncertainty of the belief. That is, the difference between the restricted model and the full model is that in the restricted model the firms have no demand uncertainty and do not update their beliefs after exporting. Note that there is heterogeneity of the demand factor across firms. At the same time, I allow the productivity of each firm to evolve as assumed by the full model. I re-solve the Bellman equation in this restricted model and average over 200 simulations. The predicted percentages of exporters in the two groups as well as all firms are shown in Table 8. The difference between the predicted percentage from the counterfactual and that from the full model tells us how export participation is affected by uncertainty and learning. The percentage of exporters, overall, decreases by 21.0%. This is intuitive, since as discussed in Section 5.2, firms have less incentive to export if there is no demand uncertainty. This result is complementary to Dickstein and Morales (2015), which measure the information set of firms at the time of their export decisions. While Dickstein and Morales (2015) test whether observable variables (observable to the researcher) were in the firm's information set, the significant decrease in this counterfactual suggests the existence of firm-level learning from unobservable demand shocks in this industry. More importantly, the percentages of exporters in potential entrants and experienced exporters decrease by 27.7% and 13.7%, respectively. The different magnitudes of decrease imply that the evolution of the demand belief has a stronger effect for potential entrants. This echoes to the finding that potential entrants face greater uncertainty compared with experienced exporters.

The second scenario is to shut down productivity evolution by eliminating the time-varying aspect of productivity. That is to set the productivity of each firm at the level when it first appears in the data set. Note that this only eliminates the productivity heterogeneity in the time dimension within a firm, but it still allows for productivity heterogeneity across firms. I also leave the demand belief to evolve as assumed by the full model. The predicted percentages of exporters are shown in Table 8. The difference between the predicted percentages from the counterfactual and full models shows how export participation is affected by productivity heterogeneity in the time dimension. The percentages of exporters in potential entrants and experienced exporters decrease by 32.1% and 23.9%, respectively, and the percentage for all firms decreases by 28.3%. The decreases are expected, because the productivity in this industry is increasing over time, thus fixing firms' productivity at the initial values generally implies lower productivity at the industrial level, and consequently less firms are able to export. Nonetheless, the key observation is that, the drop of exporting percentage of potential entrants is similar to the first scenario, while for the experienced firms the drop in this scenario is larger. This comparison implies that, productivity evolution is the major driving force of export participation for experienced exporters, but for potential entrants, demand learning plays a more important role than that of experienced firms.

The second exercise rises from the question that how important the learning process is in terms of enhancing export participation when potential entrants are promoted with lower trade cost. It is expected that the reduction of trade costs (including both fixed and sunk costs, because of various trade policies and trade liberalization) is going to boost the export participation, especially for potential entrants. The logic is that when trade cost is lower, firms with lower productivity are able to enter into foreign markets. However, the traditional analysis ignores the possibility that conditional on entry firms react to what they are learning from the markets. When entrants receive favorable demand signals and update their demand beliefs, they may continue exporting even if they are hit by adverse trade cost shocks or negative productivity shocks. This is confirmed by the finding in Section 5.2 that, the distribution of demand expectation of potential entrants moves towards right, compared with the initial belief. To show the importance of this specific role of learning, I re-slove the model and simulate dynamic export decisions of potential entrants in 2002 (the first year after China's accession to WTO) under two scenarios. In both scenarios, both fixed cost and sunk cost are reduced by ten percent permanently and firms can observe demand signals which are drawn from the distribution of the foreign demand. The difference of the two scenarios is that, in the first case firms do not react to what they observe in the foreign market when they export – firms do not learn from experience; in the other case firms update their beliefs according to the demand signals they receive – firms learn from experience. I re-solve the Bellman equation for both cases and simulated the model for 200 times. Table 9 shows that, as expected, the percentage of exporting firms increases from 8.3% to 13.5% even firms are not learning, which is a direct effect of trade cost reduction – less productive firms are able to export; however, when firms are allowed to learn from exports, this number further increases to 16.2%. This indirect effect though the learning process rises because after the initial opportunity of exporting due to the reduction of the trade costs, entrants receiving favorable demand signals continue exporting and even survive in the foreign market. The gap between the two scenarios is economically significant. This comparison suggests that the observed export boom of Chinese firms after joining WTO may be also attributed to the opportunity of learning about demand in foreign markets in addition to the direct effect of trade cost reduction. This also collaborates with Arkolakis, Papageorgiou, and Timoshenko (2015) which suggests that fixed cost subsidies to young firms can prevent early exit and thus enhance welfare by benefiting consumers through access to a larger number of varieties.

7 Conclusion

Traditional international trade literature has focused more on modeling production while growing recent papers have emphasized the demand force, especially the role of uncertain demand. In this paper, I incorporate both of them into a structural model of export dynamics to empirically quantify how productivity evolution and the demand learning process affect the firm-market level export participation separately. I utilize detailed data on firm-level production and transaction-level exports of Chinese ceramics industry to estimate the structural model. The estimated model is then used to quantitatively isolate the individual roles and evaluate the relative importance of the two processes. I find that exporting firms and non-exporting firms are different in both productivity and the demand belief. The demand belief heterogeneity is the dominant difference. Since productivity is measured from the domestic sales, this suggests that although the domestic performance is positively correlated to export participation, it is the foreign demand that serves as the main predictor of the success in the German market. From the dynamic perspective, productivity evolution is the major driving force of export participation for experienced exporters, but for potential entrants, the learning process plays a more important role than that of experienced exporters. I also find evidence that actual entrants who received favorable demand signals continued exporting and even survived in the foreign market. Based on this finding, a counterfactual exercise is conducted to evaluate the magnitude of this indirect effect of the trade cost reduction rising via demand learning. I find that the learning process drives the export probability of potential entrants in 2002 from 13.5%, which a non-learning model would predict, further to 16.2%. This implies that the observed export boom of Chinese firms after joining WTO may be also attributed to the opportunity of learning about demand in foreign markets in addition to the direct effect of trade cost reduction.

This paper demonstrates that both productivity evolution and the demand learning process contribute to the observed export dynamics at firm-market level, and shows how to utilize shipment-level exports and firm-level production data to estimate the role of each process. This further opens up directions for future research. With firm-level R&D data, one may examine how the productivity improvement is endogenously related to Bayesian learning about foreign demand. Going further, one may extend the framework to investigate how belief updating drives sequential exporting documented in Albornoz, Calvo Pardo, Corcos, and Ornelas (2012) when the learning processes in different markets are correlated.

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Appendices

Appendix A Dynamic Estimation Details

I describe the strategy to estimate the dynamic parameters by three stages, as in Rust (1987).

The first stage estimates θ_1 via the first partial likelihood. In particular, I estimate θ_1 using the data on the number of transactions and export participation of each firm in each period. To be specific, $\hat{\theta}_1$ is obtained as

$$\hat{\theta}_1 = \arg\max_{\theta_1} \sum_{i,t} \ln(\ell^1(n_{it}; \theta_1)). \tag{40}$$

Note that in this estimation, there is no need to calculate the value function.

Then, with the estimated $\hat{\theta}_1$, the second stage is to estimate θ_2 via the second partial likelihood:

$$\hat{\theta}_2 = \arg\max_{\theta_2} \sum_{i,t} \ln(\ell^2(e_{it}|s_{it}, e_{it-1}; \hat{\theta}_1, \theta_2)). \tag{41}$$

In principle, this stage requires solving the value function at each evaluation of ℓ^2, which is the main computational burden in estimating this high-dimention-state dynamic model. I follow the pioneer practice in Barwick and Pathak (2015) to use Sieve functional approximation combined with MPEC developed in Su and Judd (2012) in order to evaluate the value function of this dynamic model without solving it. The detailed algorithm is described in Appendix B.

The third stage is to use the estimated $(\hat{\theta}_1, \hat{\theta}_2)$ as an initial starting point to produce an efficient estimate of θ via the full likelihood:

$$\hat{\theta} = \arg\max_{\theta} \sum_{i,t} \ln(\ell^f(e_{it}, n_{it}|s_{it}, e_{it-1}; \theta_1, \theta_2)). \tag{42}$$

This stage also involves the internal evaluation of the value function for each evaluation of ℓ^f. This estimation yields a consistent estimator of asymptotic covariance matrix for θ, nonetheless, the estimate of θ from this stage is usually identical to the estimates from the first two stages (Rust, 1987).

Appendix B Value Function Approximation

This appendix describes the formulation used in the dynamic estimation to evaluate the value function in order to address challenges posed by the high dimension of state variables. The implementation follows the approach developed in Barwick and Pathak (2015): approximates the unknown value function using Sieves with parametric basis functions and cast the Bellman equation as a model constraint in the estimation procedure that has to be satisfied at the parameter estimates, as in in Su and Judd (2012). Throughout this appendix, I suppress the firm index i to simplify the notations.

First, I follow Rust (1987) to define the *expected* value function as:

$$EV(s_t, e_{t-1}, e_t) \equiv E_{s_{t+1}, e_t, \epsilon}[V(s_{t+1}, e_t, \epsilon)|s_t, e_{t-1}, e_t], \tag{43}$$

where $\epsilon = (\epsilon(0), \epsilon(1))$ is the vector of the trade cost shocks.

Given the type 1 extreme distribution of ϵ and with the above notation, the Bellman equation

(22) can be written as:

$$\begin{aligned} & EV(s_t, e_{t-1}, e_t) \\ = & \sum_{s_{t+1}, e_t} \ln \left\{ \sum_{e_{t+1}} \exp[u(s_{t+1}, e_t) + \beta EV(s_{t+1}, e_t, e_{t+1})] \right\} \times \Pr(s_{t+1}, e_t | s_t, e_{t-1}, e_t). \end{aligned} \tag{44}$$

With slight abuse of notation, I choose a set of M polynomial functions $\{f_m(s_t)\}_{m=1}^M$ as basis functions to approximate the unknown value function:

$$EV(s_t, e_{t-1}, e_t) \approx \sum_{m=1}^{M} b_m(e_{t-1}, e_t) f_m(s_t). \tag{45}$$

where $b_m(e_{t-1}, e_t)$ is the unknown coefficient associated with basis function m and export state (e_{t-1}, e_t). That is, I allow for a very flexible function form so that the value function evaluated at different export status (e_{t-1}, e_t) varies.

Plug (45) in to (44), I get:

$$\begin{aligned} & \sum_{m=1}^{M} b_j(e_{t-1}, e_t) f_m(s_t) \\ \approx & \sum_{s_{t+1}, e_t} \ln \left\{ \sum_{e_{t+1}} \exp[u(s_{t+1}, e_t) + \beta \sum_{m=1}^{M} b_m(e_t, e_{t+1}) f_m(s_{t+1})] \right\} \times \Pr(s_{t+1}, e_t | s_t, e_{t-1}, e_t). \end{aligned} \tag{46}$$

Barwick and Pathak (2015) point out that this equation holds approximately at all data states. I follow their approach to choose $\{b_m(e_{t-1}, e_t)\}_{m=1}^M$ to best fit this nonlinear equation in ''least-squared-residuals":

$$\begin{aligned} & \{\hat{b}_m(e_{t-1}, e_t)\}_{m=1}^M = \text{argmin}_{\{b_m(e_{t-1}, e_t)\}_{m=1}^M} \Bigg\| \sum_{m=1}^{M} b_m(e_{t-1}, e_t) f_m(s_t) - \\ & \sum_{s_{t+1}, e_t} \ln \left\{ \sum_{e_{t+1}} \exp[u(s_{t+1}, e_t) + \beta \sum_{m=1}^{M} b_m(e_t, e_{t+1}) f_m(s_{t+1})] \right\} \times \Pr(s_{t+1}, e_t | s_t, e_{t-1}, e_t) \Bigg\|_2 . \end{aligned} \tag{47}$$

Similarly, the conditional choice probability function (38) can be re-written as:

$$\begin{aligned} & \ell^2(e_{it} | s_t, e_{t-1}; \hat{\theta}_1, \theta_2) \\ = & \Pr(e_t = 1 | s_t, e_{t-1}; \theta_1, \theta_2) \\ = & \frac{\exp[u(s_{it}, e_{it-1}) + \beta \sum_{m=1}^{M} b_j(e_t, 1) f_j(s_{t+1})]}{\exp[u(s_{it}, e_{it-1}) + \beta \sum_{m=1}^{M} b_j(e_t, 1) f_j(s_{t+1})] + \exp[\beta \sum_{m=1}^{M} b_j(e_t, 0) f_j(s_{t+1})]}, \end{aligned} \tag{48}$$

Finally, the estimation of θ_2 via the Maximum Likelihood method (41) can be cast with the value function approximation as a constraint:

$$\begin{aligned} \hat{\theta}_2 = & \arg\max_{\theta_2} \sum_{i,t} \ln(\ell^2(e_{it} | s_t, e_{t-1}; \hat{\theta}_1, \theta_2)) \\ & \text{subject to} \quad (47). \end{aligned} \tag{49}$$

Figure 1: Portion of firms switched export status to Germany in each quarter after entry

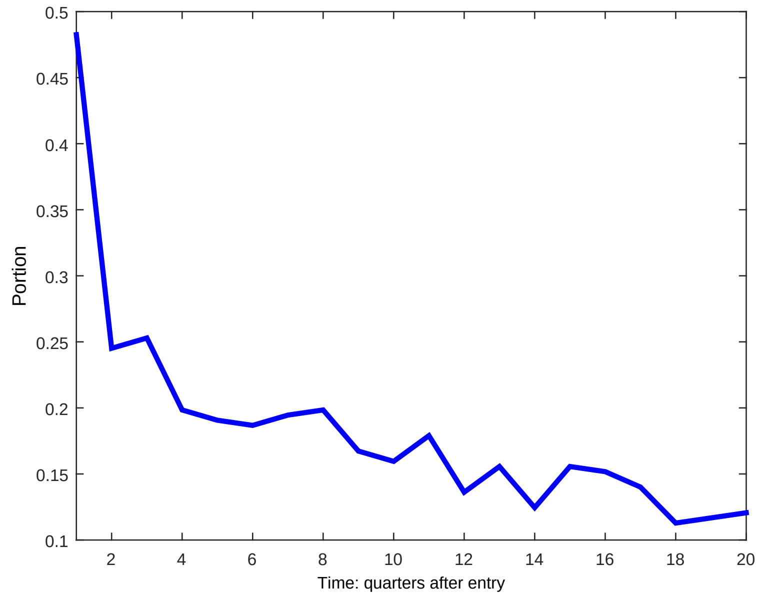

Figure 2: The resolution of demand uncertainty

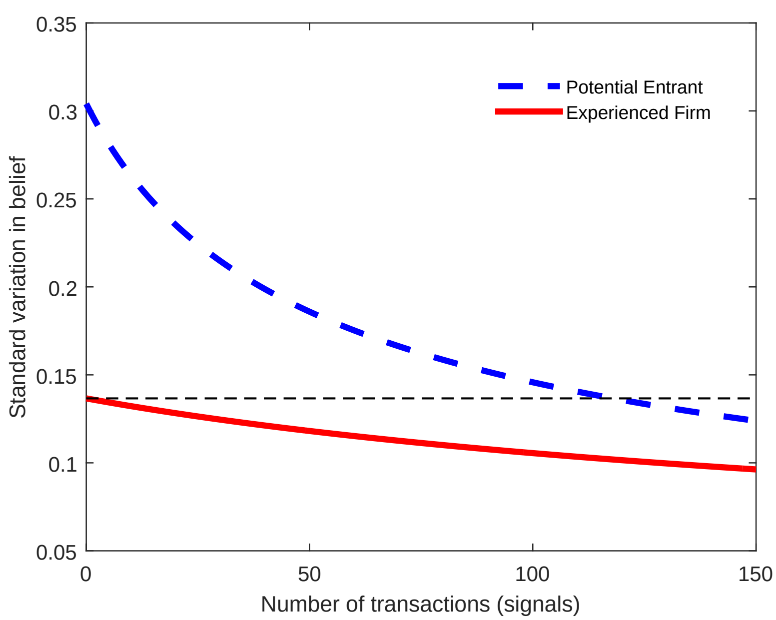

Figure 3: Productivity density by group

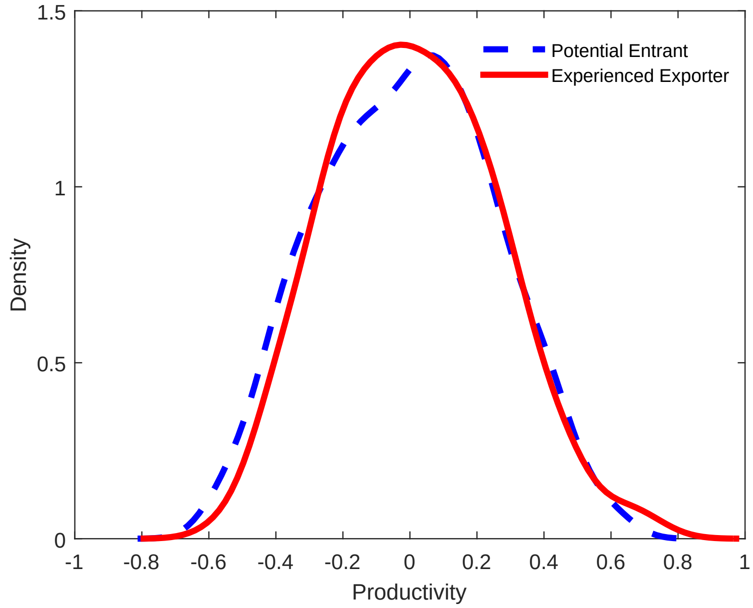

Figure 4: Expected foreign demand density by group

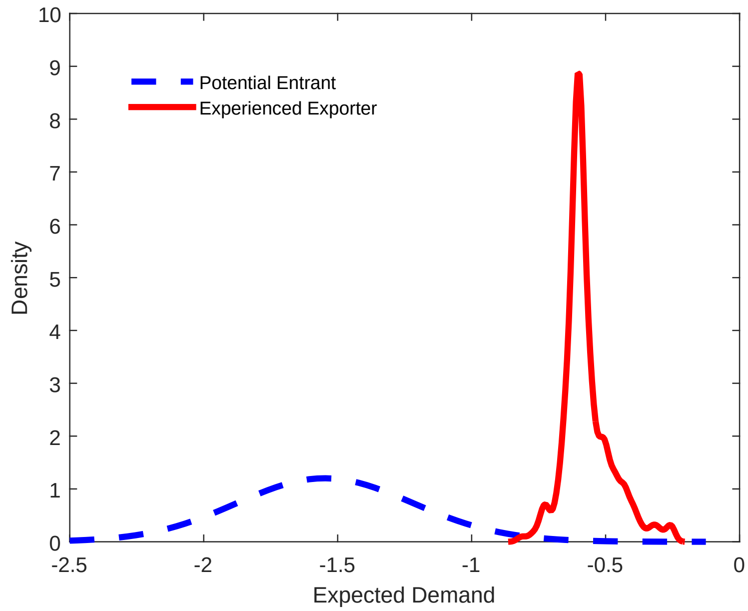

Figure 5: Initial beliefs and posterior demand distributions

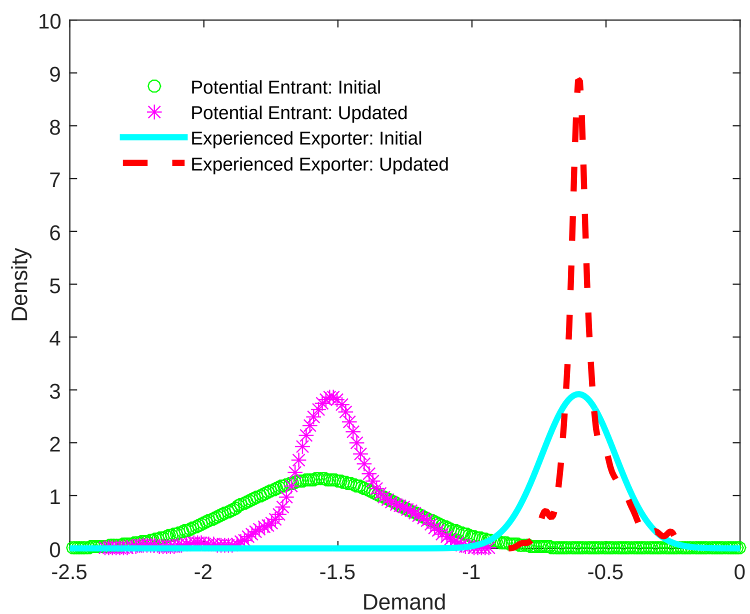

Table 1: Percentage of exporters to Germany, by entry cohort

	Year of entry			
Year	2001	2002	2003	2004
2001	100%	-	-	-
2002	65%	100%	-	-
2003	64%	66%	100%	-
2004	64%	61%	60%	100%
2005	50%	64%	60%	64%
2006	47%	61%	52%	61%

Percentages are based on the total number of firms in each cohort. For example, 50% in the 2nd column and 5th row means among firms entered in 2001, 50% of them exported in 2005.

Table 2: Estimates of linear probability model of export

Parameter	Estimate	S.E.
α_0	0.327	(0.045)
α_n	0.450	(0.104)
$\hat{\alpha}_N$	-0.153	(0.083)
R^2	0.18	
#Obs	152	

The parameters are estimated with firms in ceramic and glass industry began to export to Germany during 2002-2004. Each observation is a firm-year record.

Table 3: Estimates of marginal cost and productivity parameters

Parameter	Estimate	S.E.
γ_k	-0.073	(0.002)
g_0	0.426	(0.010)
g_1	0.844	(0.002)
R^2	0.76	
#Obs	21677	

The parameters are estimated with all firms in ceramic and glass industry from Chinese Annual Survey of Manufacturing. Each observation is a firm-year record.

Table 4: Demand elasticity estiamtes

Parameter	Estimate	S.E.
η^D	-4.175	(0.044)
η^{DEU}	-2.205	(0.496)
η^{oth}	-4.810	(0.122)
R^2	0.97	
#Obs	4978	

The parameters are estimated with firms in ceramic and glass industry that identified from both data sources. Each observation is a firm-year record.

Table 5: Estimates of dynamic parameters

	$\hat{\phi}_0$	$\hat{\phi}_n$	$\hat{m}_{00}$	$\hat{m}_{10}$	$\hat{\sigma}_{00}$	$\hat{\sigma}_{10}$	$\hat{c}^f$	$\hat{c}^s$	$\hat{\delta}$
Est.	0.752	0.706	-1.566	-0.602	0.305	0.138	0.371	2.185	0.600
S.E.	(0.079)	(0.034)	(0.008)	(0.046)	(0.072)	(0.015)	(0.037)	(0.028)	(0.022)

Table 6: Exporting and nonexporting firms: mean productivity v.s. expected demand

	$\tilde{\omega}$	$m + \sigma^2/2$
Exporting	2.576	-1.023
Non-exporting	2.544	-1.478
Profitability Difference	0.038	0.576

Table 7: Comparison in mean: potential entrants and experienced firms

	$\tilde{\omega}$	m	σ	λ
Experience Firms	2.568	-0.570	0.132	6.742
Potential Entrants	2.549	-1.523	0.296	3.584
Profitability Difference	0.024	1.593	-0.035	0.881

Table 8: Counterfactuals: learning v.s. productivity

	Percent of Exporting Firms in		
	Potential Entrants	Experienced Firms	All Firms
Full Model	14.0	80.4	23.3
No Uncertainty	10.1 (↓27.7%)	69.4 (↓13.7%)	18.4 (↓21.0%)
No Productivity Evolution	9.5 (↓32.1%)	61.2 (↓23.9%)	16.7 (↓28.3%)

Table 9: Counterfactuals: role of learning in trade cost reduction policy

Scenario of Potential Entrants in 2002	Exporting Firms (%)
Baseline (No Trade Cost Reduction and with Learning)	8.3
Trade Cost Reduction but no Learning	13.5
Trade Cost Reduction and with Learning	16.2